Math 530

Autumn 2006

Quiz 1 Solutions

(1.) We'll count ordered pairs. First, there are 10*9=90 ways to choose two balls in order, so $\sharp(\Omega) = 90$. Now we will count the number of pairs (x, y) where $1 \le x, y \le 10$ and $|x - y| \ge 2$. The easiest way to do this is count the pairs (x, y) where $|x - y| = k$ for each $k \ge 2$. We'll list in a table below the number of such pairs:

k	2	3	4	5	6	7	8	9
$\sharp$	16	14	12	10	8	6	4	2

So there are 72 such ordered pairs, so $\sharp(A) = 72$. This gives $P(A) = \frac{\sharp(A)}{\sharp(\Omega)} \frac{72}{90} = \frac{4}{5}$.

(2.) Again we will count the number of ordered 4-tuples. This gives us $\sharp(\Omega) = 10 * 9 * 8 * 7 = 5040$. Now we count all the ordered 4-tuples which have a white ball as the last ball. There are 8 "types": (w, w, w, w), (b, w, w, w), (w, b, w, w), (w, w, b, w), (w, b, b, w), (b, w, b, w), (b, b, w, w) and (b, b, b, w). There are $4*3*2*1 = 24$ of the first type; $6 * 4 * 3 * 2 = 144$ each of types 2,3,4; $6 * 5 * 4 * 3 = 180$ each of types 5,6,7; and $6 * 5 * 4 * 4$ of the last type. Adding these up, we get $\sharp(A) = 24 + 3 * 144 + 3 * 180 + 480 = 2016$. This gives:

(a.) $P(A) = \frac{\sharp(A)}{\sharp(\Omega)} = \frac{2016}{5040} = \frac{2}{5}$.

(b.) $P(B) = 1 - P(A) = \frac{3}{5}$

(3.) First we will count the number of ordered triples (x, y, z) with $1 \le x, y, z \le 20$. So $\sharp(\Omega) = 20*20*20 = 8000$

(a.) The number of ordered triples (x, y, z) with $x + y + z < 6$ is 10. So $P(A) = \frac{\sharp(A)}{\sharp(\Omega)} = \frac{10}{8000} = \frac{1}{800}$

(b.) **(a.)** The number of ordered triples (x, y, z) with $x + y + z = 6$ is 10. So $P(A) = \frac{\sharp(B)}{\sharp(\Omega)} = \frac{10}{8000} = \frac{1}{800}$

(c.) The probability that the roll sum is at most 6 is $P(A) + P(B) = \frac{1}{400}$, so the probability that the sum is greater than 6 is $1 - [P(A) + P(B)] = \frac{399}{400}$.

(4.)

(a.) $P(A \cap B) = P(A) + P(B) - P(A \cup B) = \frac{2}{5} + \frac{2}{5} - \frac{1}{3} = \frac{7}{15}$

(b.) $P(D) = P(C \cup D) + P(C \cap D) - P(C) = \frac{1}{2} + \frac{1}{4} - \frac{1}{3} = \frac{5}{12}$