Math 530

Autumn 2006

Quiz 2 Solutions

(1.) Let M_i be the event that marksman i hit the target

(a.) $P = P(M_1 \cap M_2^c \cap M_3^c) + P(M_1^c \cap M_2 \cap M_3^c) + P(M_1^c \cap M_2^c \cap M_3) = \frac{1}{3}\frac{3}{4}\frac{4}{5} + \frac{2}{3}\frac{1}{4}\frac{4}{5} + \frac{2}{3}\frac{3}{4}\frac{1}{5} = \frac{26}{60}$

(b.) $P = P(M_1 \cap M_2^c \cap M_3^c) + P(M_1^c \cap M_2 \cap M_3^c) + P(M_1^c \cap M_2^c \cap M_3) + P(M_1 \cap M_2^c \cap M_3^c) + P(M_1 \cap M_2 \cap M_3^c) + P(M_1 \cap M_2^c \cap M_3) + P(M_1^c \cap M_2 \cap M_3) + P(M_1 \cap M_2 \cap M_3) = \frac{26}{60} + \frac{1}{3}\frac{1}{4}\frac{4}{5} + \frac{1}{3}\frac{3}{4}\frac{1}{5} + \frac{2}{3}\frac{1}{4}\frac{1}{5} + \frac{1}{3}\frac{1}{4}\frac{1}{5} = \frac{3}{5}$

(c.) Let B be the event that exactly two marksmen have hit. Then

$$P(M_3^c|B) = \frac{P(B \cap M_3^c)}{P(B)} = \frac{\frac{1}{3}\frac{1}{4}\frac{4}{5}}{\frac{1}{3}\frac{1}{4}\frac{4}{5} + \frac{1}{3}\frac{3}{4}\frac{1}{5} + \frac{2}{3}\frac{1}{4}\frac{1}{5}} = \frac{4}{9}$$

(2.)

(a.) Let C_i be the event that card i is a club, S_i be the event that card i is a spade, A be the event that the third card is a club and B be the event that the first two cards are black. Note that $B = (C_1 \cap C_2) \cup (C_1 \cap S_2) \cup (S_1 \cap C_2) \cup (S_1 \cap S_2)$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{4}\frac{12}{51}\frac{11}{50} + \frac{1}{4}\frac{13}{51}\frac{12}{50} + \frac{1}{4}\frac{13}{51}\frac{12}{50} + \frac{1}{4}\frac{12}{51}\frac{13}{50}}{\frac{1}{2}\frac{25}{51}} = \frac{6}{25}$$

(b.) Let D be the event that the second card is a diamond and B be the event that the first card is black.

$$P(B|D) = \frac{P(D|B)P(B)}{P(D)} = \frac{\frac{13}{51}\frac{1}{2}}{\frac{3}{4}\frac{13}{51} + \frac{1}{4}\frac{12}{51}} = \frac{26}{51}$$

(3.) Let F_i be the event that card i is a face card

(a.)

$$P = P(F_1^c \cap F_2^c \cap F_3^c \cap F_4) = \frac{40}{52}\frac{39}{51}\frac{38}{50}\frac{12}{49} = \frac{711360}{6497400}$$

(b.) Let A be the event that more than two cards are needed, B be the event that exactly 4 cards are needed and C be the event that at most two cards are needed.

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{P(B)}{P(A)} = \frac{P(B)}{1 - P(C)} = \frac{\frac{40}{52}\frac{39}{51}\frac{38}{50}\frac{12}{49}}{1 - \frac{12}{52} - \frac{40}{52}\frac{12}{51}} = \frac{711360}{3792600}$$

(4.) Let G be the event that a person has the gene, NG the event that a person does not have the gene, $+$ be the event that the person tests positive and $-$ be the event that the person tests negative

(a.)

$$P(+) = P(+|G)P(G) + P(+|NG)P(NG) = \frac{8}{10}\frac{1}{20} + \frac{1}{10}\frac{19}{20} = \frac{27}{200}$$

(b.)

$$P(G|+) = \frac{P(+|G)P(G)}{P(+)} = \frac{\frac{8}{10}\frac{1}{20}}{\frac{8}{10}\frac{1}{20} + \frac{1}{10}\frac{19}{20}} = \frac{8}{27}$$