Math 530

Autumn 2006

Quiz 3 Solutions

(1.)

(a.) The probability that a roll is higher than 7 is $\frac{13}{20}$, so:

$$P = \binom{8}{5}\left(\frac{13}{20}\right)^5\left(\frac{7}{20}\right)^3 = \mathbf{0.2786}$$

(b.) The probability that a roll is lower than a 5 is $\frac{4}{20} = \frac{1}{5}$. The the probability that exactly k rolls are less than 5 is given by:

$$P(k) = \binom{8}{k}\left(\frac{1}{5}\right)^k\left(\frac{4}{5}\right)^{8-k}$$

So the desired probability is:

$$P = \sum_{k=0}^{4} P(k) = \mathbf{0.9896}$$

(c.) Since two 3's were rolled in the first 5 rolls, we need to find the probability that two more 3's were rolled in the last three rolls:

$$P = \binom{3}{2}\left(\frac{1}{20}\right)^2\left(\frac{19}{20}\right)^1 = \mathbf{0.0071}$$

(2.) Use the normal approximation.

(a.) $n = 1000, p = .53, q = .47$, so $\mu = np = 530$ and $\sigma = \sqrt{\mu q} = 15.783$

$$P(490 \le k \le 510) = \Phi\left(\frac{510 + 0.5 - 530}{15.783}\right) - \Phi\left(\frac{490 - 0.5 - 530}{15.783}\right)$$

$$= \Phi(-1.24) - \Phi(-2.57) = 1 - \Phi(1.24) - 1 + \Phi(2.57) = -0.8925 + 0.9949 = \mathbf{0.1024}$$

(b.) Use μ and σ from (a.)

$$P(k = 530) = \Phi\left(\frac{530 + 0.5 - 530}{15.783}\right) - \Phi\left(\frac{530 - 0.5 - 530}{15.783}\right)$$

$$= \Phi(0.03) - \Phi(-0.03) = 2\Phi(0.03) - 1 = 2(0.5160) - 1 = \mathbf{0.0320}$$

(c.) Want to solve $P\left(k \ge \frac{n}{2}\right) \ge 0.95$, this is the same as $P\left(k \le \frac{n-1}{2}\right) \le 0.05$. Note $\mu = 0.53n$ and $\sigma = \sqrt{n(0.53)(0.47)} = 0.499\sqrt{n}$.

$$P\left(k \le \frac{n-1}{2}\right) = \Phi\left(\frac{\frac{n-1}{2} + 0.5 - 0.53n}{0.499\sqrt{n}}\right) \le 0.05$$

$$\Phi\left(\frac{-0.03n}{0.499\sqrt{n}}\right) \le 0.05$$

$$1 - \Phi\left(\frac{0.03n}{0.499\sqrt{n}}\right) \le 0.05$$

$$\Phi\left(\frac{0.03n}{0.499\sqrt{n}}\right) \ge 0.95$$

$$\frac{0.03n}{0.499\sqrt{n}} \ge 1.65$$

$$\mathbf{n \ge 754}$$

(3.) We will use a Poisson approximation to find these probabilities. Here $n = 1000, p = \frac{2}{5000} = 0.0004$, so $\mu = (0.0004)(1000) = 0.4$. Let $P(k)$ be the probability of exactly k successes, then

$$P(k) = e^{-0.4}\frac{(0.4)^k}{k!}$$

(a.)

$$P = P(0) + P(1) = e^{-0.4}(1 + 0.4) = \mathbf{0.9384}$$

(b.)

$$P = P(2) = e^{-0.4}\left(\frac{(0.4)^2}{2}\right) = \mathbf{0.0536}$$

(4.)

(a.) $N = 100, G = 40, B = 60, n = 4, g = 3, b = 1$. So:

$$P = \binom{4}{3}\frac{(40)_3(60)_1}{(100)_4} = \mathbf{0.1512}$$

(b.) $N = 100, G = 60, B = 40, n = 4, g = 2, b = 2$

$$P = \binom{4}{2}\frac{(60)^2(40)^2}{(100)^4} = \mathbf{0.3456}$$