Math 530

Autumn 2006

# Quiz 4 Solutions

**(1a.)**

| Y \ X | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|
| 3 | 0 | 1/60 | 2/60 | 2/60 | 1/60 | 0 | 0 |
| 4 | 1/60 | 0 | 2/60 | 1/60 | 1/60 | 1/60 | 0 |
| 5 | 2/60 | 2/60 | 0 | 3/60 | 3/60 | 1/60 | 1/60 |
| 6 | 2/60 | 1/60 | 3/60 | 0 | 3/60 | 1/60 | 2/60 |
| 7 | 1/60 | 1/60 | 3/60 | 3/60 | 0 | 2/60 | 2/60 |
| 8 | 0 | 1/60 | 1/60 | 1/60 | 2/60 | 0 | 1/60 |
| 9 | 0 | 0 | 1/60 | 2/60 | 2/60 | 1/60 | 0 |

**(b.)** Note $4 \leq \max(X, Y) \leq 9$.

$$P(\max(X,Y) = 4) = \mathbf{\frac{2}{30}}, \qquad P(\max(X,Y) = 5) = \mathbf{\frac{8}{60}}, \qquad P(\max(X,Y) = 6) = \mathbf{\frac{12}{60}}$$

$$P(\max(X,Y) = 7) = \mathbf{\frac{16}{60}}, \qquad P(\max(X,Y) = 8) = \mathbf{\frac{10}{60}}, \qquad P(\max(X,Y) = 9) = \mathbf{\frac{12}{60}}$$

**(2a.)**

$$E(X_1 + \cdots + X_9) = 9E(X_1) = 9\left(\frac{1 + \cdots + 8}{8}\right) = \mathbf{40.5}$$

**(2b.)**

$$E(\max(X_1, \ldots, X_6)) = P(\max(X_1, \ldots, X_6) \geq 1) + \cdots + P(\max(X_1, \ldots, X_6) \geq 8)$$

$$= [1 - P(\max(X_1, \ldots, X_6) < 1)] + \cdots + [1 - P(\max(X_1, \ldots, X_6) < 8)]$$

$$= 8 - \frac{0^6}{8^6} - \frac{1^6}{8^6} - \cdots - \frac{7^6}{8^6} = \mathbf{7.2950}$$

**(c.)** Let $A_i$ be the event that face $i$ does not show up in the first 8 rolls and let $I_{A_i}$ be the indicator random variable of $A_i$. Then $I_{A_1} + \cdots I_{A_8}$ is the number of faces which do not show up in the first 8 rolls. Also $P(A_i) = \left(\frac{7}{8}\right)^8$. So:

$$E(I_{A_1} + \cdots I_{A_8}) = E(I_{A_1}) + \cdots E(I_{A_8}) = P(A_1) + \cdots + P(A_8) = 8 * \left(\frac{7}{8}\right)^8 = \mathbf{2.7489}$$

**(3.)** $P(X_i = 0) = P(X_i = 2) = \frac{1}{4}$ and $P(X_i = 1) = \frac{1}{2}$. So

$$E(X_i) = 0\frac{1}{4} + 1\frac{1}{2} + 2\frac{1}{4} = 1$$

and

$$Var(X_i) = E(X_i^2) - E(X_i)^2 = 0\frac{1}{4} + 1\frac{1}{2} + 4\frac{1}{4} - 1 = \frac{1}{2}$$

so

$$SD(X_i) = \frac{\sqrt{2}}{2}$$

Thus:

$$P(S_{100} > 90) = 1 - P(S_{100} \leq 90) \approx \Phi\left(\frac{90 - (100)(1)}{\sqrt{100}\frac{\sqrt{2}}{2}}\right) = 1 - \Phi(-1.41) = 1 - [1 - \Phi(1.41)] = \mathbf{0.9207}$$

**(4a.)**

$$P(S \leq 4) = P(X + Y = 1) + P(X + Y = 2) + P(X + Y = 3) + P(X + Y = 4) = 0 + \frac{1}{100} + \frac{2}{100} + \frac{3}{100} = \mathbf{\frac{6}{100}}$$

**(b.)**

$$P(S \leq 4 | X \leq 2) = \frac{P(X + Y \leq 4, X \leq 2)}{P(X \leq 2)} = \frac{5/100}{2/100} = \mathbf{\frac{1}{4}}$$