Math 530

Autumn 2006

Solutions to Exam 2 Review

(1a.) $h = 10$ so $P(X > 23) = 2^{-\frac{23}{10}} = \mathbf{0.2031}$

(1b.) $0.1 = P(X > t) = 2^{-\frac{t}{10}}$ so $\frac{t}{10} = \log_2(10) = 3.3226$. Thus $t = \mathbf{33.226\ years}$.

(2a.)

$$P(X = Y) = \sum_{k=0}^{\infty} P(X = k, Y = k) = \sum_{k=0}^{\infty} P(X = k)P(Y = k) = \sum_{k=0}^{\infty} e^{-\mu}\frac{\mu^k}{k!}(1-p)^k p$$
$$= e^{-\mu}p\sum_{k=0}^{\infty}\frac{\mu^k}{k!}(1-p)^k = e^{-\mu}p\sum_{k=0}^{\infty}\frac{[\mu(1-p)]^k}{k!}$$
$$= e^{-\mu}pe^{\mu(1-p)} = \mathbf{pe^{-\mu p}}$$

(2b.) $P = \frac{1}{2}e^{-1} = \mathbf{\frac{1}{2e}}$

(3a.)

$$P(Z = 1) = \binom{100}{1}\left(\frac{9}{10}\right)^{99}\left(\frac{1}{10}\right)^{1} = \mathbf{0.0003}$$

(3b.) Let A_i be the event that digit i is a zero and let I_{A_i} be the indicator random variable of A. Then $Z = I_1 + \cdots + I_{A_{100}}$.

$$E(Z) = E(I_1 + \cdots + I_{A_{100}}) = 100E(I_{A_1}) = 100P(A_1) = 100\frac{1}{10} = \mathbf{10}$$

(3c.)

$$E(S) = E(X_1 + \cdots + X_{100}) = 100E(X_1) = 100\left(\frac{0 + 1 + \cdots + 9}{10}\right) = \mathbf{450}$$

(3d.)

$$P(Z \geq 15) \leq \frac{E(Z)}{15} = \mathbf{\frac{10}{15}}$$

(3e.) $Var(Z) = 100Var(I_{A_1}) = 100[P(A_1) - E(A_1)^2] = 100[P(A_1) - P(A_1)^2] = 9$. Thus $\sigma = SD(Z) = \sqrt{9} = 3$ and $\mu = E(Z) = 10$.

$$P(Z \geq 15) = P(Z > 14) = 1 - P(Z \leq 14) \approx 1 - \Phi\left(\frac{14 - 10}{3}\right) = 1 - \Phi(1.33) = 1 - 0.9082 = \mathbf{0.0918}$$

(4a.)

$$E(S) = 25E(X) = 25\left((-1)\frac{1}{4} + (0)\frac{1}{4} + (1)\frac{1}{2}\right) = \mathbf{\frac{25}{4}}$$

(4b.) $Var(S) = 25Var(X) = 25[E(X^2) - E(X)^2] = 25\left((1)\frac{1}{4} + (0)\frac{1}{4} + (1)\frac{1}{2} - \frac{1}{16}\right) = 20.3125$

$$SD(S) = \sqrt{Var(S)} = \sqrt{20.3125} = \mathbf{4.5070}$$

(4c.)

$$P(S > 0) = 1 - P(S \leq 0) \approx 1 - \Phi\left(\frac{0 - 6.25}{4.5070}\right) = 1 - \Phi(-1.39) = \Phi(1.39) = \mathbf{0.9177}$$

(5.)

$$P(X = Y) = \sum_{k=1}^{\infty} P(X = k, Y = k) = \sum_{k=1}^{\infty} P(X = k)P(Y = k) = \sum_{k=1}^{\infty} (1 - p_1)^{k-1} p_1 (1 - p_2)^{k-1} p_2$$

$$= p_1 p_2 \sum_{j=0}^{\infty} (1 - p_1)^j (1 - p_2)^j = p_1 p_2 \sum_{j=0}^{\infty} [(1 - p_1)(1 - p_2)]^j = \frac{p_1 p_2}{1 - (1 - p_1)(1 - p_2)} = \frac{\mathbf{p_1 p_2}}{\mathbf{p_1 + p_2 - p_1 p_2}}$$

(6a.)

	X=2	3	4	5	6	7	8	9	10	11	12
Y=2	1/216	1/216	1/216	1/216	1/216	1/216	0	0	0	0	0
3	1/216	2/216	2/216	2/216	2/216	2/216	1/216	0	0	0	0
4	1/216	2/216	3/216	3/216	3/216	3/216	2/216	1/216	0	0	0
5	1/216	2/216	3/216	4/216	4/216	4/216	3/216	2/216	1/216	0	0
6	1/216	2/216	3/216	4/216	5/216	5/216	4/216	3/216	2/216	1/216	0
7	1/216	2/216	3/216	4/216	5/216	6/216	5/216	4/216	3/216	2/216	1/216
8	0	1/216	2/216	3/216	4/216	5/216	5/216	4/216	3/216	2/216	1/216
9	0	0	1/216	2/216	3/216	4/216	4/216	4/216	3/216	2/216	1/216
10	0	0	0	1/216	2/216	3/216	3/216	3/216	3/216	2/216	1/216
11	0	0	0	0	1/216	2/216	2/216	2/216	2/216	2/216	1/216
12	0	0	0	0	0	1/216	1/216	1/216	1/216	1/216	1/216

(6b.)

$$P(X = 6) = \sum_{k=2}^{12} P(X = 6, Y = k) = \frac{\mathbf{30}}{\mathbf{216}}$$

(6c.)

$$P(\min(X,Y)=12)=\frac{1}{216}$$

$$P(\min(X,Y)=11)=\frac{4}{216}$$

$$P(\min(X,Y)=10)=\frac{9}{216}$$

$$P(\min(X,Y)=9)=\frac{16}{216}$$

$$P(\min(X,Y)=8)=\frac{25}{216}$$

$$P(\min(X,Y)=7)=\frac{36}{216}$$

$$P(\min(X,Y)=6)=\frac{35}{216}$$

$$P(\min(X,Y)=5)=\frac{32}{216}$$

$$P(\min(X,Y)=4)=\frac{27}{216}$$

$$P(\min(X,Y)=3)=\frac{20}{216}$$

$$P(\min(X,Y)=2)=\frac{11}{216}$$