

Math 151A 2004 Final

Problem 1

[8 points total]

[YOU DO NOT NEED TO SHOW YOUR WORK]

For each of the following, decide whether the statement is True or False. Circle *T* for True or *F* for False.

T F The line $y = 1$ is a horizontal asymptote for the function $f(x) = \frac{3x-7}{3x^2+5}$.

T F If $f'(3) = 0$, then the tangent line to $f(x)$ at $x = 3$ is horizontal.

T F The graph of the derivative of a fifth-degree polynomial could cross the x-axis five times.

T F The period of $f(x) = \sin(3x)$ is $\frac{2\pi}{3}$.

T F Any value of x for which $f'(x) = 0$ is a local extremum of $f(x)$.

T F The Second Derivative Test can be used to check whether a given point is a local maximum or local minimum of a function $f(x)$.

T F The derivative of $f(x) = \arctan x$ is $\frac{1}{\sqrt{1+x^2}}$.

T F Suppose that $f(t)$ represents the temperature of a frozen turkey placed in an oven which has been preheated to 450° as a function of time. If the temperature of the turkey never actually reaches 450° , and if the turkey never leaves the oven, then $f''(t)$ must be negative for some t . (Hint: draw a picture.)

Problem 2

[6 points total]

[SHOW ALL WORK]

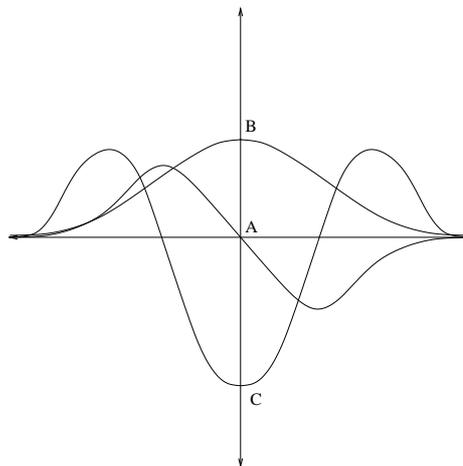
Use the **DEFINITION OF THE DERIVATIVE** to find the derivative of $f(x) = \frac{1}{x^2}$
(You will receive no credit if you use 'shortcuts'.)

Problem 3

[6 points total]

[MULTIPLE CHOICE: YOU NEED NOT SHOW YOUR WORK]

On the following axes, the graphs labelled by A, B, and C are graphs of a function f , its derivative f' , and its second derivative f'' , but not necessarily in that order. You must identify which is which.



The graph of f is ____.

The graph of f' is ____.

The graph of f'' is ____.

Problem 4

[6 points total]

[SHOW ALL WORK]

Find the equation of the tangent line at the point $(2, 1)$ to the curve whose equation is $2y^4 + x^3y^2 + 3x = 16$.

Problem 5

[10 points total]

[SHOW ALL WORK]

Let $f(x) = -x^3 + 6x^2 - 9x$.

- (a) Find each x-intercept of $f(x)$.

Answer: $x =$ _____

- (b) Determine the interval(s) on which $f(x)$ is increasing and on which $f(x)$ is decreasing.

Answer: increasing _____

Answer: decreasing _____

- (c) Find the coordinate pair of each local maximum and local minimum of $f(x)$.

Answer: local maxima, $(x, y) =$ _____

Answer: local minima, $(x, y) =$ _____

- (d) Determine the interval(s) on which $f(x)$ is concave up and on which $f(x)$ is concave down.

Answer: concave up _____

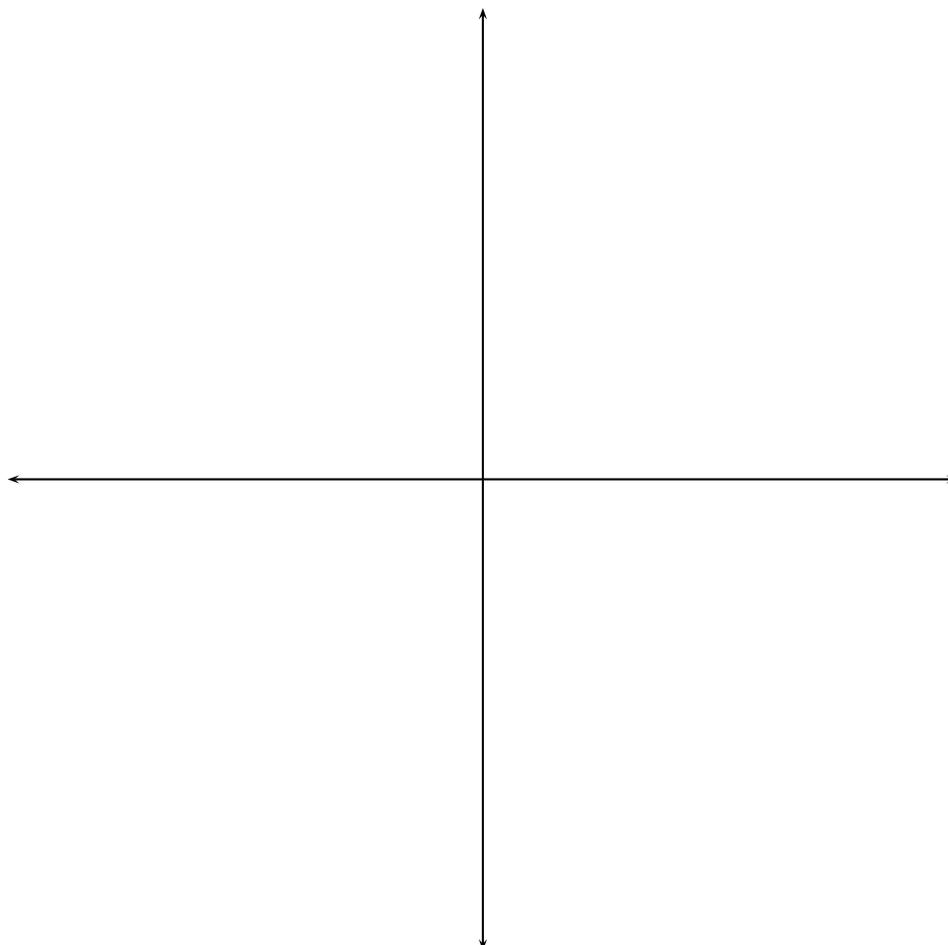
Answer: concave down _____

- (e) Find the coordinate pair of each inflection point of $f(x)$.

Answer: inflection point(s), $(x, y) =$ _____

- (f) Using (a)-(e), sketch a graph of $f(x)$ on the next page.
(Be sure to label all local extrema and inflection points.)

Coordinate Axes for Problem 5(f)



Problem 6

[8 points total]

[SHOW ALL WORK]

Atmospheric pressure, P , decreases exponentially with altitude, h . Suppose the pressure is 10 psi at an altitude of 10,000 feet and 15 psi at sea level.

- Find an equation for P , in psi, in terms of h , in thousands of feet above sea level.
- What is the pressure at an altitude of 20,000 feet?
- How many feet must you climb from your current altitude so that the atmospheric pressure is cut in half? (You may leave your answer in terms of logarithms. You do not need to come up with a decimal approximation.)
- Compute $P'(10)$. What is the practical meaning of $P'(10)$?

Problem 7

[10 points total]

[SHOW ALL WORK]

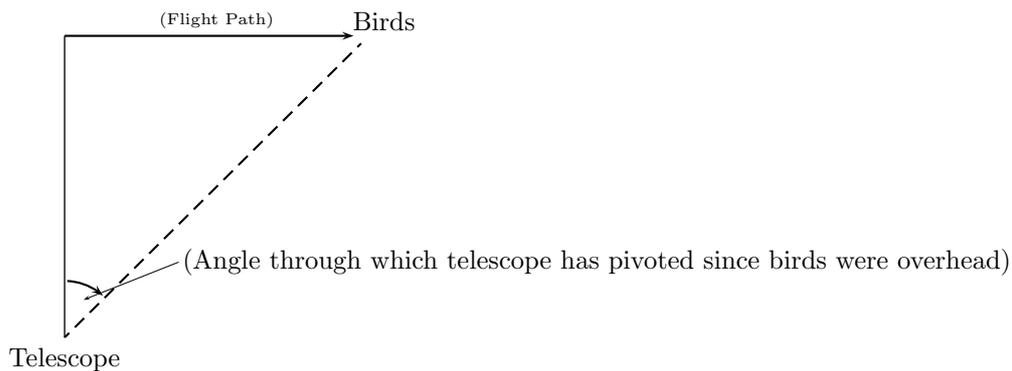
A closed rectangular box with a square base and top is to be made to contain 1250 cubic feet. The cost of the material to construct the box is \$35 per square foot for the base, \$15 per square foot for the top, and \$20 per square foot for the sides. What is the least that can be spent to construct such a box? (YOU MUST VERIFY that the cost that you find is really the least possible.)

Problem 8

[10 points total]

[SHOW ALL WORK]

A telescope is pivoting to follow the flight of a flock of birds flying due south directly overhead at a constant altitude of 40 feet and at a constant speed of 20 ft/sec. At what rate (in radians per second) is the telescope pivoting at the moment when the birds are 30 feet south of the pivot?



Problem 9

[10 points total]

[SHOW ALL WORK]

A plane is flying from an island 100 miles off a straight shore to a landing strip on that shore 400 miles to the south of take-off point. Suppose it requires three times as much fuel per mile to fly over water as over land. What flight path minimizes fuel consumption? (You may assume that the critical point gives a minimum for the fuel consumption function. Moreover, you should ignore all issues of fuel consumption caused by taking off, landing, or turning the plane. You should also ignore any effects of windspeed.)

