

## Math 151A : Calculus and Analytic Geometry I Final Exam (December 3, 2005)

Name: \_\_\_\_\_

Social Security Number: \_\_\_\_\_

Recitation Instructor: \_\_\_\_\_ Recitation Hour: \_\_\_\_\_

Please read each problem carefully, and indicate answers as directed. Your solutions **must** be supported by calculations or explanations: *no points will be given for answers that are not accompanied by, or are not consistent with, supporting work. Partial credit can be earned for steps that make reasonable progress towards a solution.*

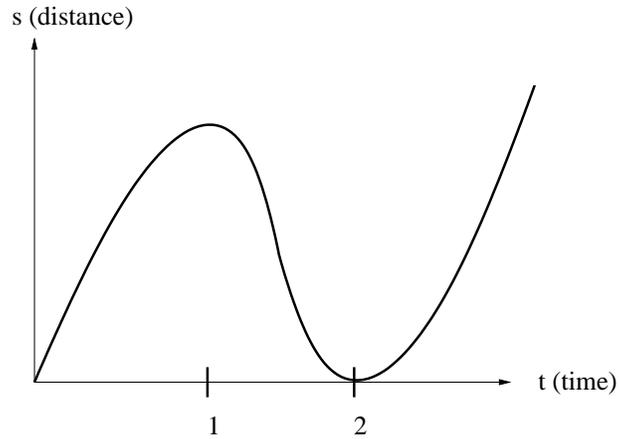
Problem #	Points	Score
1	8	
2	8	
3	12	
4	8	
5	8	
6	14	
7	8	
8	8	
9	6	
Total	80	

1. Suppose  $x$  and  $y$  satisfy  $x^3 + 2xy + y^2 = 4$ .

(a) [2 points] Find the coordinates of all the points on the curve with  $x = 1$ .

(b) [6 points] Find the equation of the tangent line to the curve above at each point from part (a).

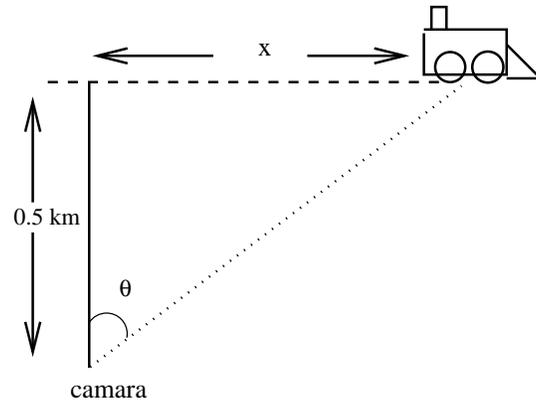
2. A particle is moving along a straight line; its acceleration is zero only once. Its distance,  $s$ , to the right of its starting point is given by the figure below. Estimate:



- (a) [3 points] The time interval(s) over which the particle is moving to the right, and when it is moving to the left.

- (b) [5 points] Estimate when the acceleration of the particle is zero, when it is negative, and when it is positive.

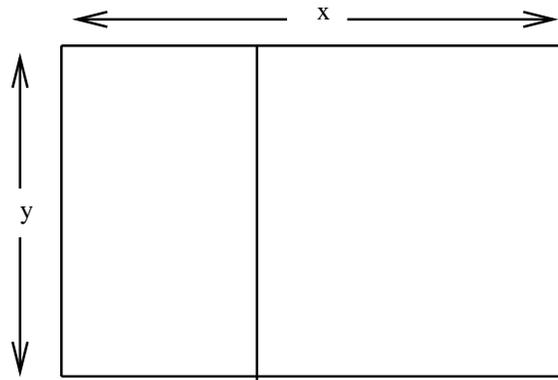
3. A train is moving at 2 km/min along a straight track, due east. A movie camera, 0.5 km away from the track, is focused on the train.



- (a) [6 points] How fast is the distance from the camera to the train changing when the train is 1 km from the camera? Give units.

- (b) [6 points] How fast is the camera rotating (in radians/min) at the moment when the train is 1 km from the camera?

4. [8 points] A farmer is building two rectangular pens, side-by-side, as pictured below. If the outer fencing material costs \$5 per foot, the inner fencing (for the divider) is \$80 per foot, and she must enclose 100 square feet total, what is the minimum she can spend?



5. [8 points] Find the derivative of  $f(x) = \frac{1}{x^2}$  using the definition of the derivative (NO DERIVATIVE RULES ALLOWED - i.e. no shortcuts!)

6. Let  $f(x) = -x^3 + 6x^2 - 9x$ .

(a) [1 point] Find each  $x$ -intercept of  $f(x)$ .

*Answer:*  $x =$  \_\_\_\_\_

(b) [4 points] Determine the interval(s) on which  $f(x)$  is increasing and on which  $f(x)$  is decreasing.

*Answer:* increasing \_\_\_\_\_

*Answer:* decreasing \_\_\_\_\_

(c) [1 point] Find the coordinate pair of each local maximum and local minimum of  $f(x)$ .

*Answer:* local maxima,  $(x, y) =$  \_\_\_\_\_

*Answer:* local minima,  $(x, y) =$  \_\_\_\_\_

(d) [4 points] Determine the interval(s) on which  $f(x)$  is concave up and on which  $f(x)$  is concave down.

*Answer:* concave up \_\_\_\_\_

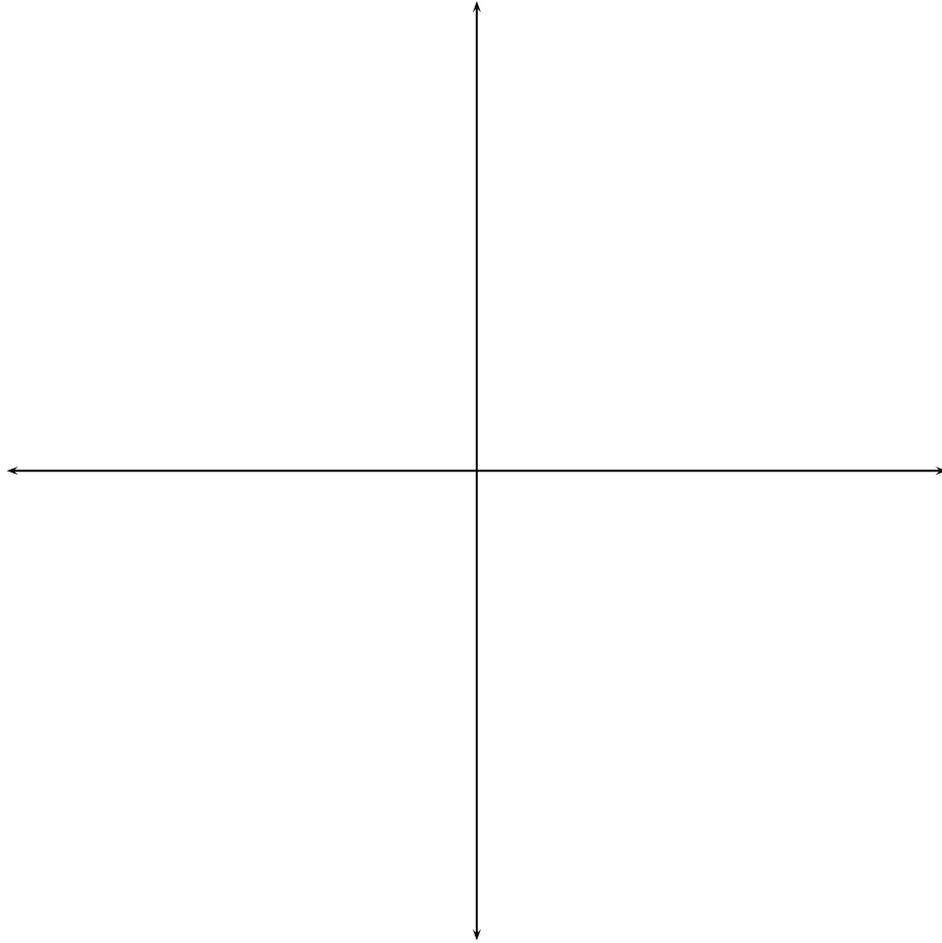
*Answer:* concave down \_\_\_\_\_

(e) [1 point] Find the coordinate pair of each inflection point of  $f(x)$ .

*Answer:* inflection point(s),  $(x, y) =$  \_\_\_\_\_

(f) [3 points] Using (a)-(e), sketch a graph of  $f(x)$  on the next page.  
(Be sure to label all local extrema and inflection points.)

Coordinate Axes for Problem 6(f)

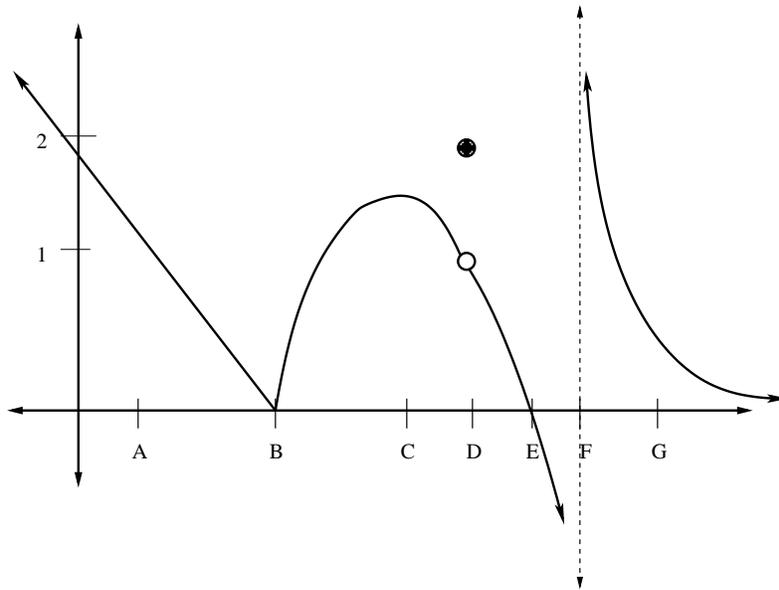


7. [8 points] The total cost of producing  $q$  goods is given by:

$$C(q) = -q^3 + 45q^2 + 100q$$

If each item is sold for \$700, how many items should you produce to maximize the profit  $P(q)$ ?

8. Answer the questions below using the following graph for  $f(x)$ :



(a) [2 points] At what values of  $x$  is  $f$  not continuous?

(b) [2 points] Is  $f$  differentiable everywhere? If not, at what values of  $x$  does  $f$  not have a derivative?

(c) [2 points] Does  $\lim_{x \rightarrow D} f(x)$  exist? If so, what is its value?

(d) [2 points] Does  $\lim_{x \rightarrow F} f(x)$  exist? If so, what is its value?

9. Let  $k$  be a constant, and define  $f$  to be

$$f(x) = \begin{cases} kx, & \text{if } 0 \leq x \leq 2 \\ 3x^2, & \text{if } x > 2 \end{cases}$$

(a) [2 points] Find  $\lim_{x \rightarrow 2^+} f(x)$ .

(b) [2 points] Find  $\lim_{x \rightarrow 2^-} f(x)$ .

(c) [2 points] Is there a value of  $k$  so that  $\lim_{x \rightarrow 2} f(x)$  exists?