Review 1

(1.) Consider the following system of linear equations:

$$\begin{aligned} 2x_1 - 3x_2 + 4x_3 + 2x_4 + x_5 &= 12 \\ 3x_1 - 4x_2 - 2x_3 + x_4 &= 1 \\ 4x_1 + 5x_2 + x_3 - 6x_4 &= 21 \\ x_1 - x_2 - 2x_3 - x_4 + 3x_5 &= 1 \\ 2x_1 - 2x_2 + x_4 - x_5 &= 0 \end{aligned}$$

(a.) Find the augmented matrix of this system.
(b.) Find the reduced echelon form of the matrix in (a.).
(c.) Is this system consistent? If so, find all solutions.

(2.) Let $A = \begin{pmatrix} 1 & 3 & 1 & 1 & 0 \\ 0 & 0 & 2 & 1 & 3 \\ -1 & -2 & 1 & 1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 2 & 2 \\ -1 & 1 & 1 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 2 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{pmatrix}$.

Find AB, BA, BC, $A + C$ and $B + C$.

(3.) Given the augmented matrix A=$\left(\begin{array}{ccc|c} 1 & -1 & 3 & 1 \\ 3 & 2 & -2 & -2 \\ -2 & 6 & \alpha & \beta \end{array}\right)$, find all values of α and β such that the corresponding linear system has:

(a.) No solutions.
(b.) A unique solution.
(c.) Infinitely many solutions.

(4.) Find an elementary matrix $\mathcal{E}$ such that $\mathcal{E}A = B$ where:

(a.) $A = \begin{pmatrix} 2 & -2 & 3 \\ -1 & 4 & 2 \\ 3 & 1 & 2 \end{pmatrix}$ $B = \begin{pmatrix} 2 & -2 & 3 \\ 3 & -12 & -6 \\ 3 & 1 & 2 \end{pmatrix}$ **(b.)** $A = \begin{pmatrix} 3 & 1 & 4 \\ 1 & 2 & 3 \\ 5 & 3 & 1 \end{pmatrix}$ $B = \begin{pmatrix} 3 & 1 & 4 \\ 1 & 2 & 3 \\ 7 & 7 & 7 \end{pmatrix}$

(5.) True or False. You do not need to explain your answer.

(a.) If A and B are $n \times n$ matrices, then $\det(A + B)=\det(A)+\det(B)$
(b.) If A and B are $n \times n$ matrices, then $AB = BA$
(c.) Suppose $A\mathbf{x} = \mathbf{b_1}$ has a unique solution, then it is possible for $A\mathbf{x} = \mathbf{b_2}$ has more than one solution.
(d.) An underdetermined system of linear equations is always consistent.
(e.) Every $n \times n$ elementary matrix is invertible.

(6.) Find the inverse and determinant of the given matrices:

(a.) $\begin{pmatrix} 2 & 5 & -2 \\ 1 & -1 & 4 \\ 3 & 0 & 1 \end{pmatrix}$ **(b.)** $\begin{pmatrix} 1 & 3 & 1 & 2 \\ -1 & 2 & 0 & 1 \\ 0 & 5 & -1 & -1 \\ 0 & -1 & 2 & 0 \end{pmatrix}$

(7.) Prove the following:

(a.) If A is invertible, then $\det(A^{-1})=1/\det(A)$
(b.) If A is invertible, then $A\mathbf{x} = \mathbf{b}$ has a unique solution for each $\mathbf{b}$
(c.) If A is invertible, then $A = \mathcal{E}_1\mathcal{E}_2 \cdots \mathcal{E}_k$ where $\mathcal{E}_1, \mathcal{E}_2, \ldots \mathcal{E}_k$ are elementary matrices.