Math 151

Spring 2004

Exam 3 Review

(1.) For the given function $f(x)$ and the given interval I, find:

> **(i.)** All critical points of $f(x)$ on I.
>
> **(ii.)** All subintervals of I on which $f(x)$ is increasing and on which $f(x)$ is decreasing.
>
> **(iii.)** Where all relative maximums and relative minimums of $f(x)$ on I occur.
>
> **(iv.)** The extreme values for $f(x)$ on I.

(a.) $f(x) = x^3+3x^2-9x+10$ and $I = (-\infty, \infty)$

(b.) $f(x) = \frac{1}{x^3+1}$ and $I = (-\infty, \infty)$

(c.) $f(x) = x^3 - x^2$ and $I = [-1, 1]$

(d.) $f(x) = x^4+2x^3$ and $I = (-1, 4]$

(e.) $f(x) = \frac{x-1}{x^2+1}$ and $I = [-5, 5]$

(f.) $f(x) = \frac{x}{x^2+1}$ and $I = (-3, 3]$

(2.) Find the intervals on which $f(x)$ is concave up and on which $f(x)$ is concave down. Also, find all inflection points of $f(x)$.

(a.) $f(x) = x^3 - x^2$ **(b.)** $f(x) = \frac{1}{1+3x^2}$

(3.) The product of two negative numbers is 36. What is their largest possible sum? What is their smallest possible sum?

(4.) Consider the triangle with vertices $(1, 0)$, $(0, 1)$ and $(0, -1)$ and a point P with coordinates $(x, 0)$ with $0 \leq x \leq 1$ (Note: P lies on the x-axis and is inside the triangle). For what value of x is the sum of the distances from P to the three vertices minimal?

(5.) Suppose that a cylindrical container with a top is to be constructed which must have a volume of 100in^3. Find the least expensive container to produce if the cost of the material for the top is 1 dollar per square inch, the cost of side material is 2 dollars per square inch and the cost of the bottom material is 5 dollars per square inch.

(6.) Suppose it costs $1 + 3x^2$ dollars to ship x items by plane and $5 + x^2$ dollars to ship x items by truck. If 200 items are to be shipped, how many should be sent by each method in order for the cost to be minimal?

(7.) Find the indefinite integral:

(a.) $\int (x+1)^3 \, dx$ **(b.)** $\int (x-1)(x+2) \, dx$

(8.) Find $f(x)$ given that:

(a.) $f''(x) = x, f(1) = 1$ and $f(-1) = 2$

(b.) $f''(x) = 2, f(0) = 2$ and $f(2) = 0$