Review 1

(1.) Consider the following system of linear equations:

$$\begin{aligned}3x_1 - 2x_2 + 4x_3 + 3x_4 - x_5 &= 17\\ 2x_1 + 4x_3 + x_4 + 2x_5 &= 8\\ -2x_1 + 2x_2 - 2x_3 - x_4 + 2x_5 &= -10\\ -2x_1 - 2x_2 - 6x_3 + 2x_4 - 6x_5 &= 0\end{aligned}$$

(a.) Find the augmented matrix of this system.
(b.) Find the reduced row echelon form of the matrix in (a.).
(c.) Is this system consistent? If so, find all solutions.

(2.) Given the augmented matrix A=$\left(\begin{array}{ccc|c} 1 & -1 & 3 & 1\\ 3 & 2 & -2 & -2\\ -2 & 7 & \alpha & \beta\end{array}\right)$, find all values of α and β such that the corresponding linear system has:
(a.) No solutions.
(b.) A unique solution.
(c.) Infinitely many solutions.

(3.) Find an elementary matrix $\mathcal{E}$ such that $\mathcal{E}A = B$ where:

(a.) $A = \begin{pmatrix} 2 & -2 & 3\\ -1 & 4 & 2\\ 3 & 1 & 2\end{pmatrix}$ $B = \begin{pmatrix} 2 & -2 & 3\\ 3 & -12 & -6\\ 3 & 1 & 2\end{pmatrix}$ **(b.)** $A = \begin{pmatrix} 3 & 1 & 4\\ 1 & 2 & 3\\ 5 & 3 & 1\end{pmatrix}$ $B = \begin{pmatrix} 3 & 1 & 4\\ 1 & 2 & 3\\ 7 & 7 & 7\end{pmatrix}$

(4.) Suppose $A = \begin{pmatrix} 2 & 1 & 3\\ 0 & 0 & 7\\ 4 & 0 & 11\end{pmatrix}$ and $B = \begin{pmatrix} 2 & 1 & 3\\ 0 & -2 & 5\\ 0 & 0 & 1\end{pmatrix}$

(a.) Find three elementary matrices $\mathcal{E}_1$, $\mathcal{E}_2$, and $\mathcal{E}_3$ such that $\mathcal{E}_3\mathcal{E}_2\mathcal{E}_1 A = B$.
(b.) Compute $\det(A)$ using the results of part (a.). DO NOT USE MATLAB.

(5.) Find an $L - U$ factorization for $A = \begin{pmatrix} 1 & 3 & 5\\ 2 & 4 & 7\\ -2 & 4 & 1\end{pmatrix}$.

(6.) Find the inverse and determinant of the given matrices:

(a.) $\begin{pmatrix} 2 & 5 & -2\\ 1 & -1 & 4\\ 3 & 0 & 1\end{pmatrix}$ **(b.)** $\begin{pmatrix} 1 & 3 & 1 & 2\\ -1 & 2 & 0 & 1\\ 0 & 5 & -1 & -1\\ 0 & -1 & 2 & 0\end{pmatrix}$

(7.) True or False. You do not need to explain your answer.

(a.) If A and B are $n \times n$ matrices, then $\det(A + B)$=$\det(A)$+$\det(B)$.
(b.) If A and B are $n \times n$ matrices, then $AB = BA$.
(c.) Suppose $A\mathbf{x} = \mathbf{b_1}$ has a unique solution, then it is possible for $A\mathbf{x} = \mathbf{b_2}$ to have more than one solution.
(d.) An underdetermined system of linear equations is always consistent.
(e.) Every $n \times n$ elementary matrix is invertible.
(f.) If $\mathbf{x}_p$ is a solution to $A\mathbf{x} = \mathbf{b}$ and $\mathbf{x}_h$ is a solution to $A\mathbf{x} = \mathbf{0}$, then $\mathbf{x}_p + \mathbf{x}_h$ is also a solution to $A\mathbf{x} = \mathbf{b}$.

(8.) Prove the following:

(a.) If A is invertible, then $\det(A^{-1}) = \det(A)^{-1}$.
(b.) If A is invertible, then $A\mathbf{x} = \mathbf{b}$ has a unique solution for each $\mathbf{b}$.
(c.) If A is invertible, then $A = \mathcal{E}_1\mathcal{E}_2 \cdots \mathcal{E}_k$ where $\mathcal{E}_1, \mathcal{E}_2, \ldots \mathcal{E}_k$ are some elementary matrices.