Review 3

(1.) Suppose that $f(x) = \begin{cases} -1 & -\pi \leq x < 0 \\ 1 & 0 \leq x \leq \pi \end{cases}$. Find the nth order Fourier series approximation of $f(x)$ over $[-\pi, \pi]$.

(2.) Let $\mathbf{q}_1(x) = 1$, $\mathbf{q}_2(x) = 2x - 1$ and $\mathbf{q}_3(x) = 12x^2 - 12x + 2$ be polynomials in $C[0,1]$ and let $S = \text{Span}(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3)$.

(a.) Show that $\{\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3\}$ form an orthogonal basis for S with respect to the usual inner product on $C[0,1]$.

(b.) Find the least squares approximation of x^3 in S.

(3.) To measure the takeoff performance of an airplane, the horizontal position of the plane was measured every second from $t = 0$ to $t = 5$. The positions in meters were: $0, 29.9, 104.7, 222.0, 380.4$ and 571.7

(a.) Find the least squares cubic curve $y = \beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 t^3$ for these data.

(b.) Use the result of (a.) to estimate the velocity of the plane when $t = 4.5$ seconds.

(4.) Let $A = \begin{pmatrix} 7 & -6 & -4 & 1 \\ -5 & 1 & 0 & -2 \\ 10 & 11 & 7 & -3 \\ 19 & 9 & 7 & 1 \end{pmatrix}$

(a.) Find the condition number of A with respect to the 1-norm on A.

(b.) Find the condition number of A with respect to the ∞-norm on A.

(c.) Is this matrix well-conditioned or ill-conditioned? Explain your answer.

(5.) Suppose that $A = \begin{pmatrix} -6 & 3 & -27 & -33 & -13 \\ 6 & -5 & 25 & 28 & 14 \\ 8 & -6 & 34 & 38 & 18 \\ 12 & -10 & 50 & 41 & 23 \\ 14 & -21 & 49 & 29 & 33 \end{pmatrix}$.

(a.) Find a basis for $\text{Col}(A)$.

(b.) Find a basis for $\text{Col}(A)^{\perp}$.

(6.) True or False:

(a.) If A is an $m \times n$ matrix, then $\dim(\text{Col}(A)) + \dim(\text{N}(A^T)) = n$.

(b.) If $\mathbf{p}$ is the orthogonal projection of $\mathbf{x}$ onto the subspace S, then $\mathbf{p}$ and $\mathbf{p} + \mathbf{x}$ are orthogonal.

(c.) The matrix equation $A\mathbf{x} = \mathbf{b}$ has a unique least squares solution $\hat{\mathbf{x}}$.

(d.) If V is a normed linear space and $\mathbf{x}$ and $\mathbf{y}$ are vectors in V, then $||\mathbf{x} + \mathbf{y}|| \geq ||\mathbf{x}|| + ||\mathbf{y}||$.

(e.) If V is a inner product space and S is a finite dimensional subspace of V, then S has an orthonormal basis.

(7.) Prove:

(a.) If S is subspace of the inner product space V, then the orthogonal complement $S^{\perp}$ of S is also a subspace.

(b.) If $\mathbf{x}$ and $\mathbf{y}$ are orthogonal vectors in the inner product space V, then $||\mathbf{x} - \mathbf{y}||^2 = ||\mathbf{x}||^2 + ||\mathbf{y}||^2$.

(c.) If $\mathbf{p}$ is the projection of $\mathbf{x}$ onto the line spanned by $\mathbf{y}$, then $\mathbf{p}$ is orthogonal to $\mathbf{x} - \mathbf{p}$.