Math 571

Spring 2005

Quiz 4

Instructions: This quiz is worth a total of 40 points, and the value of each question is listed with each question. You may use any notes or books but you must work individually. The only computation aid which you may use is MATLAB, unless otherwise indicated. Make sure to write clearly and justify your answers.

(1.)(12 pts.) Determine if the given vectors are linearly independent in V where:

(a.) $\{(3,-1,-4)^T,(3,-3,-1)^T,(12,-2,-19)^T\}$ in $V=\mathbb{R}^3$.

(b.) $\{x^2+1,x-1,x+2\}$ in $V=\mathbb{P}_3$.

(c.) $\{e^{2x},e^{2(x-1)}\}$ in $V=C[-1,1]$.

(d.) $\{\sin mx,\cos nx\}$ in $V=C[-\pi,\pi]$.

(2.)(12 pts.) Find a basis for the subspace S of V where:

(a.) $S=\text{Span}((-3,-2,0,2)^T,(-5,2,-4,8)^T,(-2,-4,2,-1)^T,(1,-2,2,-3)^T)$ and $V=\mathbb{R}^4$

(b.) $S=\{p(x)\in\mathbb{P}_4 \mid p(1)=0 \text{ and } p(-1)=0\}$ and $V=\mathbb{P}_4$

(c.) $S=\{A\in\mathbb{R}^{3\times 3} \mid A \text{ is upper triangular }\}$ and $V=\mathbb{R}^{3\times 3}$

(d.) $S=\text{Span}(1,\cos^2 x,\cos 2x\}$ and $V=C[-\pi,\pi]$

(3.) In the following problems, $\mathcal{A}=\{A\in\mathbb{R}^{3\times 3} \mid A^T=-A\}$ and $\mathcal{S}=\{S\in\mathbb{R}^{3\times 3} \mid S^T=S\}$.

(a.)(6 pts.) Show that $\mathcal{A}$ and $\mathcal{S}$ are subspaces of $\mathbb{R}^{3\times 3}$.

(b.)(6 pts.) Find a basis $\{A_1,\ldots,A_m\}$ for $\mathcal{A}$ and a basis $\{S_1,\ldots,S_n\}$ for $\mathcal{S}$. (Hint: write down a typical vector for each space.)

(c.)(2 pts.) Show that if $A+S=\mathbf{0}$ for any A in $\mathcal{A}$ and S in $\mathcal{S}$, then $A=S=\mathbf{0}$. (Hint: take the transpose of this sum.)

(d.)(2 pts.) Show that $\{A_1,\ldots,A_m,S_1,\ldots,S_n\}$ is a basis for $\mathcal{A}+\mathcal{S}$. (Hint: you only need to show that these vectors are linearly independent.)

Extra Credit:(3 pts.) In (3.), show that $\mathcal{A}+\mathcal{S}=\mathbb{R}^{3\times 3}$.