

Quiz 5

**Instructions:** This quiz is worth a total of 40 points, and the value of each question is listed with each question. You may use any notes or books but you must work individually. The only computation aid which you may use is MATLAB, unless otherwise indicated. Make sure to write clearly and justify your answers.

(1.)(8 pts.) Determine if the subset  $S$  of the vector space  $V$  is a subspace where:

- (a.)  $V = C^2[-1, 2]$  and  $S$  is the set of functions  $f(x)$  such that  $f''(0) \geq 0$ .
- (b.)  $V = C^2[-1, 2]$  and  $S$  is the set of functions  $f(x)$  such that  $f''(x) - 3f'(x) + 2f(x) = 0$ .
- (c.)  $V = \mathbb{R}^3$  and  $S = \{(x, y, z)^T \mid x + 2y = z^2\}$
- (d.)  $V = \mathbb{R}^{4 \times 4}$  and  $S$  is the set of  $4 \times 4$  non-singular matrices.

(2.)(20 pts.) For the given vector space  $V$  and the subset  $S$  do the following:

- (i.) Show that  $S$  is a subspace of  $V$ .
- (ii.) Find a basis for  $S$ .
- (iii.) Determine  $\dim(S)$ .

- (a.)  $V = \mathbb{R}^{2 \times 2}$  and  $S$  is the set of  $2 \times 2$  matrices  $A = (a_{ij})$  with  $a_{11} + a_{22} = 0$ .
- (b.)  $V = \mathbb{P}_4$  and  $S$  is the set of polynomials  $p(x)$  in  $V$  such that  $p'(1) = 0$
- (c.)  $V = \mathbb{R}^{3 \times 3}$  and  $S$  is the set of  $3 \times 3$  upper triangular matrices  $A$  with  $A^3 = \mathbf{0}$ . (hint: Find a typical vector first.)

(d.)  $V = \mathbb{R}^5$  and  $S = N(A)$  where  $A = \begin{pmatrix} 1 & 2 & -1 & 0 & 5 \\ 2 & -1 & 1 & 1 & 4 \\ 0 & 0 & 2 & -1 & -2 \end{pmatrix}$

(3.)(12 pts.) Suppose that the charge in a system is distributed amongst 4 poles labeled  $P_1, P_2, P_3$  and  $P_4$ . Whenever the system receives a pulse of light of a certain wavelength, the individual electrons may redistribute themselves with the following probabilities. (The probabilities are listed in parentheses.)

$P_1 \rightarrow P_2; (0.32)$	$P_1 \rightarrow P_3; (0.30)$	$P_1 \rightarrow P_4; (0.28)$
$P_2 \rightarrow P_1; (0.34)$	$P_2 \rightarrow P_3; (0.11)$	$P_2 \rightarrow P_4; (0.08)$
$P_3 \rightarrow P_1; (0.25)$	$P_3 \rightarrow P_2; (0.12)$	$P_3 \rightarrow P_4; (0.42)$
$P_4 \rightarrow P_1; (0.33)$	$P_4 \rightarrow P_2; (0.05)$	$P_4 \rightarrow P_3; (0.30)$

- (a.) Find the transition matrix of this Markov process.
- (b.) If the initial distribution is given by the vector  $(100, 30, 45, 50)^T$ , find the distribution after 2 pulses and after 100 pulses.
- (c.) What is the steady-state probability vector of this process?