

### Project 3

**Instructions:** This project is worth a total of 25 points. You may use any notes or books that you wish but you must work individually. The only computation aid which you may use is MATLAB, unless otherwise indicated. The primary reference for this project is the notes on Markov processes and the notes on graphs which can be found at: <http://www.math.ohio-state.edu/~husen/teaching/571/markov.pdf> and <http://www.math.ohio-state.edu/~husen/teaching/571/graphs.pdf> Make sure to write clearly and justify your answers.

(1.)(8 pts.) Suppose that one type of atom can be in one of three quantum states which are denoted  $Q_1$ ,  $Q_2$  and  $Q_3$ . Given a collection of these atoms, it is found that each measurement perturbs the atoms and each atom has a chance of switching its quantum state. The probabilities that they do are given by:

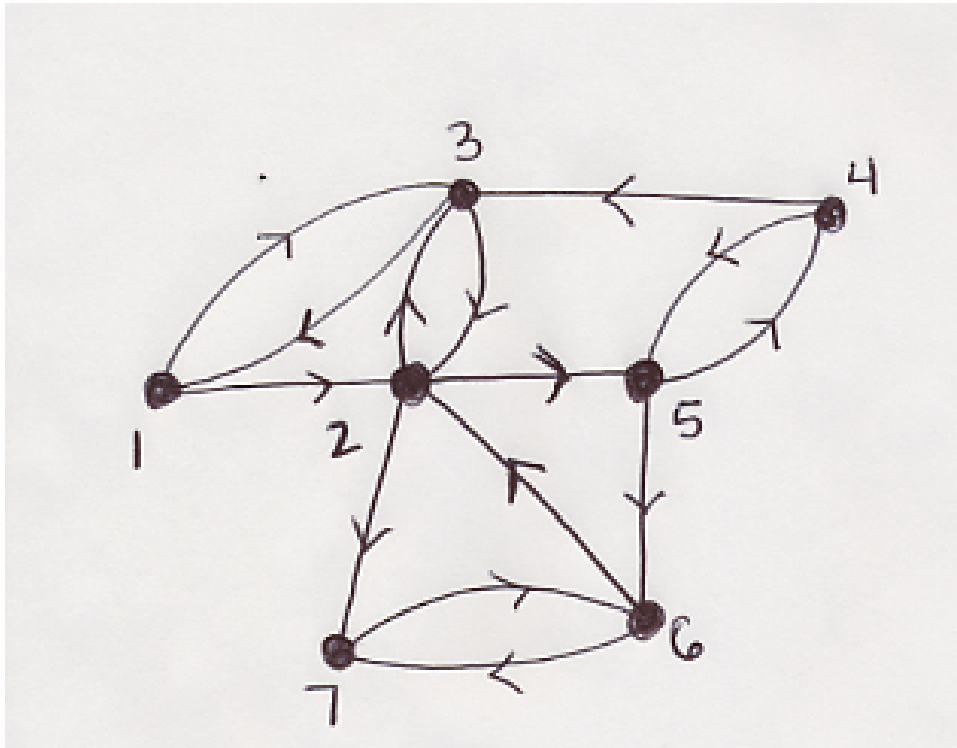
$$\begin{array}{ll} Q_1 \rightarrow Q_2; P = 0.55 & Q_1 \rightarrow Q_3; P = 0.10 \\ Q_2 \rightarrow Q_1; P = 0.3 & Q_2 \rightarrow Q_3; P = 0.25 \\ Q_3 \rightarrow Q_1; P = 0.45 & Q_3 \rightarrow Q_2; P = 0.20 \end{array}$$

- (a.) Find the transition matrix  $M$  of this Markov process.
- (b.) Suppose that initially there are 100 atoms in state  $Q_1$ , 75 in state  $Q_2$  and 50 in state  $Q_3$ . What is the resulting distribution after 5 measurements?
- (c.) Find the steady-state probability vector  $\mathbf{x}_s$  of this process.

(2.)(8 pts.) Suppose that one type of atom can exist in a ground state  $G$  or be in one of 4 excited states  $E_1$ ,  $E_2$ ,  $E_3$  and  $E_4$ . Suppose that a collection of these atoms is irradiated by a photon pulse. If an atom is in the ground state, then it has a probability of 0.35 of being excited to state  $E_1$ , a probability of 0.2 of being excited to state  $E_2$  and it never transitions to  $E_3$  or  $E_4$ . If it is in state  $E_1$  it has a probability of 0.45 of dropping to the ground state, a probability of 0.2 of being excited to state  $E_3$ , a probability of 0.2 of being excited to state  $E_4$  and it never transitions to  $E_2$ . If it is in state  $E_2$  it has a probability of 0.7 of dropping to the ground state and it never transitions to  $E_1$ ,  $E_3$  or  $E_4$ . If it is in state  $E_3$  or  $E_4$  it has a 0.85 probability of dropping to state  $E_1$  and a 0.15 probability of remaining in its current state.

- (a.) Find the transition matrix  $M$  of this Markov process.
- (b.) Find the steady-state probability vector  $\mathbf{x}_s$  of this process.
- (c.) How many bursts will it take for this process to be within 4 decimal places of accuracy of the steady-state?

(3.)(9 pts.) Consider the digraph below:



- (a.) Find the adjacency matrix of this digraph.
- (b.) Find the transition matrix associated to a random walk on this digraph.
- (c.) Suppose than an object starts at vertex 2. What is the most likely vertex for this object to be at after 7 random walks on this digraph? Indicate the probability of this happening.
- (d.) What is the steady-state distribution of objects moving randomly on this digraph?