Math 571

Spring 2007

Project 3

Instructions: This project is worth a total of 25 points. You may use any notes or books that you wish but you must work individually. The only computation aid which you may use is MATLAB, unless otherwise indicated. The primary reference for this project is the notes on Markov processes and the notes on graphs which can be found at: `http://www.math.ohio-state.edu/~husen/teaching/571/markov.pdf` and `http://www.math.ohio-state.edu/~husen/teaching/571/graphs.pdf` Make sure to write clearly and justify your answers.

(1.)(8 pts.) Suppose that one type of atom can be in one of three quantum states which are denoted Q_1, Q_2 and Q_3. Given a collection of these atoms, it is found that each measurement perturbs the atoms and each atom has a chance of switching its quantum state. The probabilities that they do are given by:

$$Q_1 \to Q_2; P = 0.45 \qquad Q_1 \to Q_3; P = 0.20$$
$$Q_2 \to Q_1; P = 0.2 \qquad Q_2 \to Q_3; P = 0.25$$
$$Q_3 \to Q_1; P = 0.45 \qquad Q_3 \to Q_2; P = 0.30$$

(a.) Find the transition matrix M of this Markov process.

(b.) Suppose that initially there are 100 atoms in state Q_1, 75 in state Q_2 and 25 in state Q_3. What is the resulting distribution after 5 measurements?

(c.) Find the steady-state probability vector $\mathbf{x}_s$ of this process.

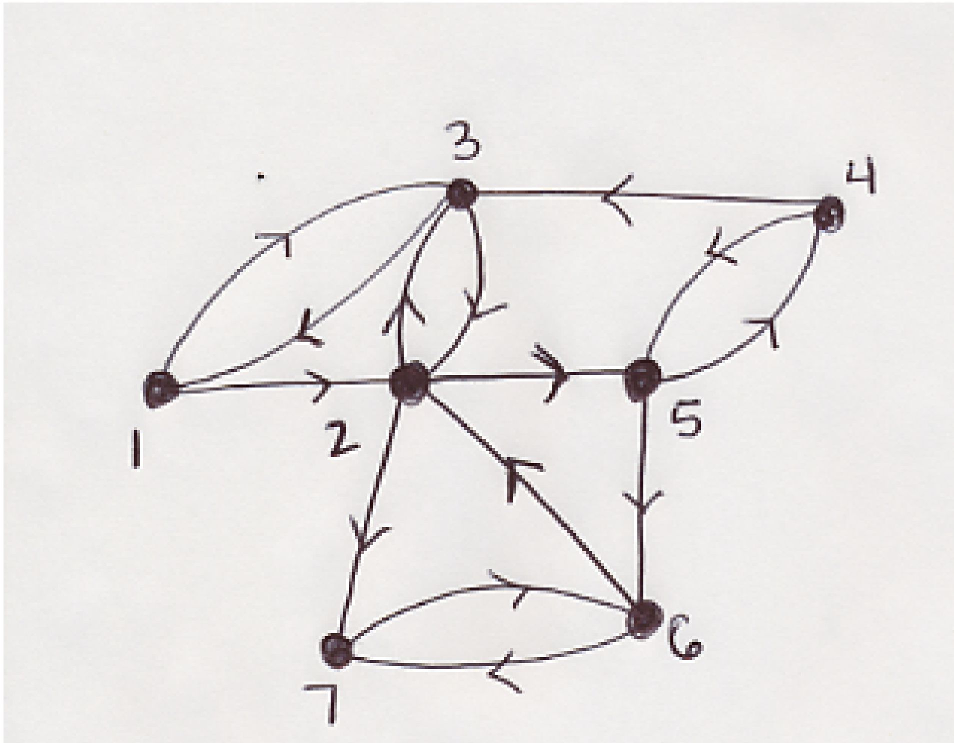
(2.)(8 pts.) Suppose that one type of atom can exist in a ground state G or be in one of 4 excited states E_1, E_2, E_3 and E_4. Suppose that a collection of these atoms is irradiated by a photon pulse. If an atom is in the ground state, then it has a probability of 0.45 of being excited to state E_1, a probability of 0.2 of being excited to state E_2 and it never transitions to E_3 or E_4. If it is in state E_1 it has a probability of 0.45 of dropping to the ground state, a probability of 0.2 of being excited to state E_3, a probability of 0.2 of being excited to state E_4 and it never transitions to E_2. If it is in state E_2 it has a probability of 0.8 of dropping to the ground state and it never transitions to E_1, E_3 or E_4. If it is in state E_3 or E_4 it has a 0.85 probability of dropping to state E_1 and a 0.15 probability or remaining in its current state.

(a.) Find the transition matrix M of this Markov process.

(b.) Find the steady-state probability vector $\mathbf{x}_s$ of this process.

(c.) How many bursts will it take for this process to be within 4 decimal places of accuracy of the steady-state?

(3.)(9 pts.) Consider the digraph below:



(a.) Find the adjacency matrix of this digraph.

(b.) Find the transition matrix associated to a random walk on this digraph.

(c.) Suppose than an object starts at vertex 3. What is the most likely vertex for this object to be at after 8 random walks on this digraph? Indicate the probability of this happening.

(d.) What is the steady-state distribution of objects moving randomly on this digraph?