

Quiz 3

Instructions: This quiz is worth 44 points and the point value for each question is listed with each question. You may use any notes or books but you must work individually. The only computation aid which you may use is MATLAB, unless otherwise indicated. Make sure to write clearly and justify your answers.

(1.)(15 pts.) Determine if the given subset S of the vector space V is a subspace of V :

- (a.) $V = \mathbb{R}^3$ and $S = \{(x_1, x_2, x_3)^T \in V \mid x_3 = x_1 + x_2\}$
- (b.) $V = \mathbb{R}^{n \times n}$ and $S = \{A \in V \mid A = A^T\}$
- (c.) $V = \mathbb{P}_4$ and $S = \{p(x) \in V \mid p(x) \text{ has at least one real root}\}$
- (d.) $V = C[-1, 1]$ and $S = \{f(x) \in V \mid f(-1) = -f(1)\}$
- (e.) $V = \mathbb{R}^{2 \times 2}$, A is a fixed matrix in V and $S = \{B \in V \mid AB = BA\}$

(2.)(6 pts.) Determine if S is a spanning set for V :

- (a.) $V = \mathbb{R}^3$ and $S = \{(2, 1, -2)^T, (3, 2, -2)^T, (2, 2, 0)^T\}$
- (b.) $V = \mathbb{P}_3$ and $S = \{2, x^2, x, 2x + 3\}$

(3.)15 pts.) Determine if the vectors in S are linearly independent in V :

- (a.) $V = \mathbb{R}^3$ and $S = \{(2, 1, -2)^T, (3, 2, -2)^T, (2, 2, 0)^T\}$
- (b.) $V = C[0, 1]$ and $S = \{1, e^x + e^{-x}, e^x - e^{-x}\}$
- (c.) $V = C[-\pi, \pi]$ and $S = \{1, \cos(2x), \sin^2(x)\}$
- (d.) $V = [0, 1]$ and $S = \{2x, |x|\}$
- (e.) $V = [-1, 1]$ and $S = \{2x, |x|\}$

(4.)(8 pts.) Prove the following:

(a.) Let $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_m$ be linearly independent vectors in \mathbb{R}^n . If A is a non-singular $n \times n$ matrix, then $A\mathbf{x}_1, A\mathbf{x}_2, \dots, A\mathbf{x}_m$ are also linearly independent vectors in \mathbb{R}^n .

(b.) If $\{\mathbf{v}_1, \dots, \mathbf{v}_n\}$ are linearly independent vectors in V which do not span V , then there is a vector \mathbf{v} in V such that $\{\mathbf{v}_1, \dots, \mathbf{v}_n, \mathbf{v}\}$ is linearly independent.