Math 571

Winter 2005

Quiz 4

Instructions: Each question is worth 4 points for a total of 16 points. You may use any notes or books but you must work individually. The only computation aid which you may use is MATLAB, unless otherwise indicated. Make sure to write clearly and justify your answers.

Suppose that U and V are subspaces of the vector space W. Furthermore suppose that $U \cap V = \{\mathbf{0}\}$. The direct sum of U and V is the set:

$$U \oplus V = \{\mathbf{u} + \mathbf{v} \mid \mathbf{u} \in U \text{ and } \mathbf{v} \in V\}$$

(1.) Prove that $U \oplus V$ is a subspace of W. (Hint: this is similar to a homework question.)

(2.) Show that if $\mathbf{u} + \mathbf{v} = \mathbf{0}$ then $\mathbf{u}$ and $\mathbf{v}$ are elements of $U \cap V$, hence are equal to $\mathbf{0}$. (Hint: if $\mathbf{u} + \mathbf{v} = \mathbf{0}$ then $\mathbf{u} = -\mathbf{v}$)

(3.) Show that if $\{\mathbf{u}_1, \ldots, \mathbf{u}_m\}$ is a basis for U and $\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ is a basis for V then $\{\mathbf{u}_1, \ldots, \mathbf{u}_m, \mathbf{v}_1, \ldots, \mathbf{v}_n\}$ is a basis for $U \oplus V$. (Hint: It is immediate that they span, just show they are linearly independent using (2.))

(4.) Show that $\dim(U \oplus V) = \dim(U) + \dim(V)$.