

Quiz 1

Instructions: Each question is worth 10 points. You may use any notes or books but you must work individually. The only computation aid which you may use is MATLAB, unless otherwise indicated. Make sure to write clearly and justify your answers.

(1.) Consider the following linear system:

$$\begin{aligned}x_1 + 2x_2 + x_3 &= 5 \\2x_1 + 7x_2 + 8x_3 &= 11 \\5x_1 + 4x_2 + \alpha x_3 &= \beta\end{aligned}$$

- (a.) Write down the augmented matrix of this system.
- (b.) Find the values of α and β such that this system has:
 - (i.) no solution,
 - (ii.) a unique solution,
 - (iii.) infinitely many solutions.

(2.) Consider the following linear system:

$$\begin{aligned}x_1 + 3x_2 + x_3 + x_4 &= 3 \\-2x_1 + 2x_2 + x_3 + 2x_4 &= 8 \\3x_1 + x_2 + 2x_3 - x_4 &= -1\end{aligned}$$

- (a.) Write down the augmented matrix of this system.
 - (b.) Put the matrix from part (a.) into reduced row echelon form. DO NOT USE MATLAB
 - (c.) Find the general solution to this linear system.
 - (d.) Find the parametric solution to this linear system.
- (3.) Suppose that A is the coefficient matrix of some linear system and U is some echelon form of A .
- (a.) Show that if the last row of U is a non-zero row, then the system must be consistent.
 - (b.) Show that if the last row of U is a zero row, then the system might be inconsistent. (hint: give an example)
- (4.) Suppose that A is the coefficient matrix of an $n \times n$ linear system. What must the reduced row echelon form of A look like if this system has no free variables? Explain.