Math 571

Winter 2006

Quiz 5

Instructions: This quiz is worth 40 points and the value of of each question is listed with each question. You may use any notes or books but you must work individually. The only computation aid which you may use is MATLAB, unless otherwise indicated. Make sure to write clearly and justify your answers.

(1.)(10 pts.) Let $\langle \mathbf{x}, \mathbf{y} \rangle = \mathbf{x}^T * \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} * \mathbf{y}$ where $\mathbf{x}, \mathbf{y}$ are vectors in $\mathbb{R}^3$ and $*$ is the usual matrix multiplication.

(a.) Show that this defines an inner product on $\mathbb{R}^3$.

(b.) If $\mathbf{x} = (1, 2, 1)^T$ and $\mathbf{y} = (-1, 2, 2)^T$, find:

- **(i.)** $\langle \mathbf{x}, \mathbf{y} \rangle$,
- **(ii.)** $||\mathbf{x}||$ and $||\mathbf{y}||$,
- **(iii.)** the orthogonal projection of $\mathbf{x}$ onto $\mathbf{y}$ and
- **(iv.)** the angle between $\mathbf{x}$ and $\mathbf{y}$.

(2.)(6 pts.) Let S be the subspace of $\mathbb{R}^4$ with $S = \text{Span}\{(1, 1, 2, -1)^T, (-4, 1, 6, -2)^T\}$. Find a basis for $S^\perp$.

(3.)(8 pts.) Suppose that $A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 2 \\ 3 & 1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} -3 & 3 & 2 \\ -4 & 1 & 1 \\ 1 & -2 & -2 \end{pmatrix}$. Find

(a.) $\langle A, B \rangle$

(b.) $||A||$

(c.) $||B||$

(d.) The angle between A and B

(e.) The orthogonal projection of A onto the space spanned by B.

(4.)(8 pts.) Let $\langle f(x), g(x) \rangle$ be the usual inner product on $C[0, 1]$. If $f(x) = x^2 + 1$ and $g(x) = 3x$, find:

(a.) $\langle f(x), g(x) \rangle$

(b.) $||f(x)||$

(c.) $||g(x)||$

(d.) The angle between $f(x)$ and $g(x)$

(e.) The orthogonal projection of $f(x)$ onto the space spanned by $g(x)$.

(5.)(8 pts.) Let $A = \begin{pmatrix} 3 & 6 & 3 & 3 & 6 & 1 \\ 5 & 10 & 4 & 2 & -3 & 6 \\ 7 & 14 & 2 & 0 & -11 & 7 \\ 1 & 2 & 0 & 0 & -1 & 2 \end{pmatrix}$ and find:

(a.) A basis for $\text{Col}(A^T)$

(b.) A basis for $\text{Col}(A^T)^\perp$.