Review 2

(1.) Let $A = \begin{pmatrix} 3 & 6 & 3 & 3 & 6 & 1 \\ 5 & 10 & 4 & 2 & -3 & 6 \\ 7 & 14 & 2 & 0 & -11 & 7 \\ 1 & 2 & 0 & 0 & -1 & 2 \end{pmatrix}$ and find:

(a.) A basis for the row space of A.
(b.) A basis for the column space of A.
(c.) A basis for the nullspace of A. DO NOT USE MATLAB.
(d.) The rank of A.

(2.) Suppose that $\mathbf{v}_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$, $\mathbf{v}_2 = \begin{pmatrix} 0 \\ -1 \\ 3 \end{pmatrix}$, $\mathbf{v}_3 = \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}$ and $\mathbf{v}_4 = \begin{pmatrix} 2 \\ 2 \\ 8 \end{pmatrix}$. Find a basis for Span$(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4)$.
Note: there is more than one answer.

(3.) Determine if S is a subspace of V where:
(a.) $V = \mathbb{R}^{2\times 2}$ and S is the set of 2×2 matrices A with $\det(A)=0$.
(b.) $V = \mathbb{R}^{2\times 2}$ and S is the set of 2×2 upper triangular matrices.
(c.) $V = \mathbb{R}^2$ and $S = \{(x_1, x_2)^T \mid |x_1| = |x_2|\}$.
(d.) $V = \mathbb{P}_5$ and S is the set of all polynomials $p(x)$ in V such that $p(1) = 0$.
(e.) $V = C[-1, 1]$ and S is the set of odd functions in V.

(4.) Determine if $1, e^x$ and $\cos x$ are linearly independent in $C[0, 1]$.

(5.) Find a basis for the subspace S of V where:
(a.) $V = \mathbb{R}^4$ and $S = \{(a - b + c, a + c, a + 2b - c, b - 3c)^T \mid a, b, c$ are real numbers $\}$.
(b.) $V = C[0, 1]$ and S=Span$(1, \sin 2x, \sin x \cos x)$.
(c.) $V = \mathbb{P}_4$ and S is the set of all polynomials $p(x)$ in V with $p(0) = 0$ and $p(1) = 0$.

(6.) Let $A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ and let $S = \{B \in \mathbb{R}^{2\times 2} \mid AB = BA\}$.

(a.) Show that S is a subspace of $\mathbb{R}^{2\times 2}$.
(b.) Find a basis for S.

(7.) True or False?
(a.) A linearly independent set can not contain $\mathbf{0}$.
(b.) A subspace of a vector space must contain $\mathbf{0}$.
(c.) If A is an $m \times n$ matrix, then dim(Col(A))+dim(N(A))=m.
(d.) If A is an $m \times n$ matrix, then $A\mathbf{x} = \mathbf{b}$ is consistent if and only if $\mathbf{b}$ is in the column space of A.
(e.) If A is a singular $n \times n$ matrix, then the columns of A form a basis for $\mathbb{R}^n$.
(f.) A spanning set can never be linearly independent.

(8.) Prove the following:
(a.) Let S be the subset of $\mathbb{R}^{n\times n}$ consisting of matrices A such that $A^T = A$. Show that S is a subspace of $\mathbb{R}^{n\times n}$. (The matrices in S are called symmetric matrices.)
(b.) If $\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ are linearly independent vectors in a vector space V, then $\{\mathbf{v}_2, \ldots, \mathbf{v}_n\}$ do not span V.
(c.) If $\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ are linearly independent vectors in a vector space V which do not span V, then there is a vector $\mathbf{v}_0$ in V such that $\{\mathbf{v}_0, \mathbf{v}_1, \ldots, \mathbf{v}_n\}$ is also linearly independent.