

Project 2

Instructions: This project is worth a total of 50 points. You may use any notes or books that you wish but you must work individually. The only computation aid which you may use is MATLAB, unless otherwise indicated. The primary reference for this project is the notes on random walks on graphs which can be found at: <http://www.math.ohio-state.edu/husen/teaching/571/graphs.pdf> Make sure to write clearly and justify your answers. WRITE ANY PROBABILITY AS A FRACTION, NOT AS A DECIMAL.

(1.)(10 pts.) Find a graph or digraph which has the given matrix as an adjacency matrix:

$$\begin{array}{l}
 \text{(a.)} \begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix} \\
 \text{(b.)} \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}
 \end{array}$$

(2.)(20 pts.) For the graph in Figure 1, find:

- (a.) The adjacency matrix of this graph,
- (b.) The probability that a random walk of length 5 begins at vertex 2,
- (c.) The probability that a random walk of length 7 begins at vertex 3 and ends at vertex 1,
- (d.) The probability that a random walk of length 6 ends at vertex 4,
- (e.) The probability that a random walk of length 8 which starts at vertex 5 will end at vertex 6.

(3.)(20 pts.) For the digraph in Figure 2, find:

- (a.) The adjacency matrix of this digraph,
- (b.) The probability that a random walk of length 10 begins at vertex 2 and ends at vertex 7,
- (c.) The probability that a random walk of length 6 ends at vertex 1,
- (d.) The probability that a random walk of length 8 which ends at vertex 6 started at vertex 3,
- (e.) The probability that a random walk of length 5 begins at vertex 2.

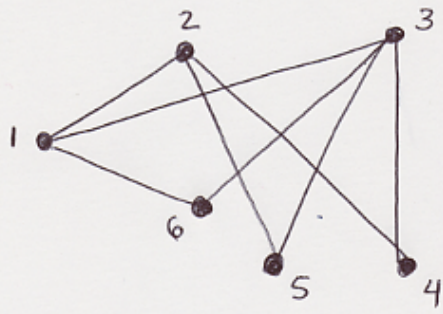


FIG. 1.

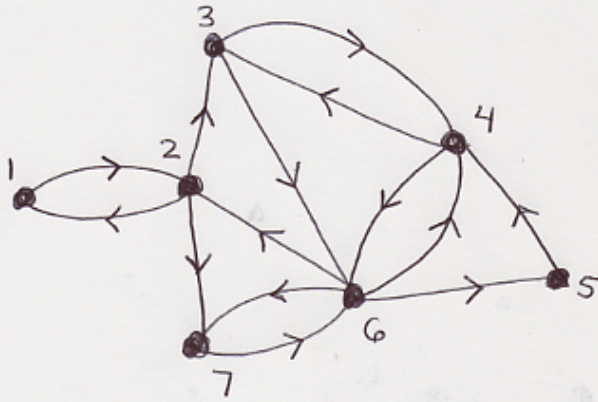


FIG. 2.