

Project 4

Instructions: This project is worth a total of 50 points. You may use any notes or books that you wish but you must work individually. The only computation aid which you may use is MATLAB, unless otherwise indicated. The primary reference for this project is the notes on inner product spaces which can be found at: <http://www.math.ohio-state.edu/~husen/teaching/571/ch5.pdf>

Make sure to write clearly and justify your answers.

(1.)(10 pts.) Find ALL least square solutions to the system:

$$\begin{aligned}x_1 + 4x_2 + 5x_3 + x_4 &= 1 \\3x_1 + 7x_2 + 10x_3 - x_4 &= 1 \\2x_1 + 3x_2 + 5x_3 + 2x_4 &= 2 \\5x_1 + 6x_2 + 11x_3 + x_4 &= 1 \\x_1 + 2x_2 + 3x_3 &= -1\end{aligned}$$

(2.)To measure the takeoff performance of an airplane, the horizontal position of the plane was measured every second from $t = 0$ to $t = 5$. The positions in meters were: 0, 29.9, 104.7, 222.0, 380.4 and 571.7

(a.)(8 pts.) Find the least squares cubic curve $y = \beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 t^3$ for these data.

(b.)(2 pts.) Use the result of (a.) to estimate the velocity of the plane when $t = 4.5$ seconds

(3.) It is known theoretically that the binding energy BE of an atom with a symmetric nucleus and an atomic mass number of A is given by:

$$BE = a_1 A - a_2 A^{2/3} - a_3 A^{5/3}$$

The values for a_1, a_2 and a_3 must be found experimentally. Four experiments were conducted and the following results were recorded for (A, BE) : (12, 92.163MeV), (20, 160.645MeV), (28, 236.526MeV) and (36, 306.719MeV).

(a.)(8 pts.) Find the best approximation for BE using least squares.

(b.)(2 pts.) Estimate the atomic mass number (to the nearest integer) which maximizes the binding energy. (Note: this value will be much higher than the true value)

(4.)(10 pts.) Suppose that $f(x) = \begin{cases} -1 & -\pi \leq x < 0 \\ 1 & 0 \leq x \leq \pi \end{cases}$. Find the n th-order Fourier series for $f(x)$ over $[-\pi, \pi]$

(5.) Let $f(x) = x^2$.

(a.)(8 pts.) Find the 10th-order Fourier series for $f(x)$.

(b.)(2 pts.) Use the result in (a.) to give an estimate for $\frac{\pi^2}{12}$. (Hint: take $x = 0$ in (a.))