Math 571
Winter 2008

Some Notes on the Final Exam

(1.) Know how to solve a system of linear equations.
- **(a.)** How to form the augmented matrix.
- **(b.)** How to row reduce by hand.
- **(c.)** How to use MATLAB to row reduce.
- **(d.)** Understand the number of solutions that a linear system has.
- **(e.)** How to find the general solution.

(2.) Know what an elementary matrix is.
- **(a.)** How does multiplication by an elementary matrix correspond to row reduction.
- **(b.)** How to find the inverse of a matrix.
- **(c.)** How to use elementary matrices to find a determinant.

(3.) Know the algebra of matrices
- **(a.)** Scalar multiplication, matrix addition, and matrix multiplication.
- **(b.)** Transposes of matrices.

(4.) Understand vector spaces.
- **(a.)** Know the primary examples of vector spaces: $\mathbb{R}^n$, $\mathbb{R}^{m\times n}$, $\mathbb{P}_n$ and $C[a,b]$.
- **(b.)** What a subspace is and how to determine if a subset is a subspace
- **(c.)** How to tell if a set of vectors spans a subspace.
- **(d.)** How to tell if a set of vectors is linearly independent.
- **(e.)** How to compute and use a Wronskian.
- **(f.)** How to find a basis and determine the dimension of a subspace.
- **(g.)** Know what the columnspace, rowspace and nullspace of a matrix is and how to find a basis for each.

(5.) Understand inner product spaces.
- **(a.)** Know the standard inner products on our primary examples of vector spaces.
- **(b.)** How to determine if a function is an inner product.
- **(c.)** How to compute the inner products, norms, angles between and orthogonal projections of vectors with a given inner product.
- **(d.)** Know what an orthogonal and an orthonormal basis is.
- **(e.)** How to compute the orthogonal projection of a vector onto a subspace.
- **(f.)** Know the relationship between the columnspace of a matrix and the nullspace of its transpose.

(6.) Understand the method of least squares.
- **(a.)** Know that the least square approximation is just the orthogonal projection onto a subspace.
- **(b.)** Know how to find all the least square solutions to a linear system.
- **(c.)** Know how to find the nth order Fourier series of a function.

(7.) Understand norms and condition numbers.
- **(a.)** How to determine if a given function is a norm.
- **(b.)** Know the primary examples of norms on $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$.
- **(c.)** How to compute the norm of a vector or matrix with a given norm.
- **(d.)** How to find the condition number of a matrix.
- **(e.)** How to find an upper an lower bound of the relative error in some calculation

(8.) Know the basic MATLAB commands to do the above.