Math 571

Winter 2008

Quiz 6

Instructions: This quiz is worth 10 points and the value of of each question is listed with each question. You may use any notes or books but you must work individually. The only computation aid which you may use is MATLAB, unless otherwise indicated. Make sure to write clearly and justify your answers.

(1.)(2 pts.) Let S be the subspace of $\mathbb{R}^5$ spanned by $\mathbf{v}_1 = (1, 5, -1, 3, 2)^T, \mathbf{v}_2 = (3, 3, 1, 1, -10)^T$ and $\mathbf{v}_3 = (7, 5, 13, -9, 4)^T$.

(a.) Verify that $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is an orthogonal basis for S.

(b.) Find the orthogonal projection $\mathbf{p}$ of $\mathbf{u} = (2, 1, 2, 1, 2)^T$ onto S.

(c.) Find the norm of the residual vector $\mathbf{u} - \mathbf{p}$ where $\mathbf{u}$ and $\mathbf{p}$ are as in (b.).

(2.)(3 pts.)Let V be the vector space $C[-1, 1]$ with the usual inner product and let S be the subspace of V spanned by $\mathbf{v}_1 = 1, \mathbf{v}_2 = x$ and $\mathbf{v}_3 = 3x^2 - 1$

(a.) Verify that $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is an orthogonal basis for S.

(b.) Find the orthogonal projection $\mathbf{p}$ of $\mathbf{u} = \sqrt[3]{x}$ onto S.

(c.) Find the norm of the residual vector $\mathbf{u} - \mathbf{p}$ where $\mathbf{u}$ and $\mathbf{p}$ are as in (b.).

(3.)(1 pts.) Find the least squares solution $\hat{\mathbf{x}}$ to the system: $\begin{pmatrix} 1 & 4 & 6 \\ 1 & 2 & 2 \\ 2 & 1 & 1 \\ -1 & -1 & 1 \\ 3 & 3 & 5 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 2 \\ 1 \\ -3 \\ 1 \\ 2 \end{pmatrix}$

(4.)(2 pts.) Suppose that a system is modeled by a function of the form $f(x) = a + b\sin(x) + c\cos(x)$. The following four data points $(x, f(x))$ are collected: $(0, -12), (\frac{\pi}{4}, 2), (\frac{5\pi}{12}, \sqrt{6})$ and $(\frac{7\pi}{6}, 2\sqrt{3})$.

(a.) Find the best approximation of $f(x)$ using the method of least squares.

(b.) Find the approximate value of $f(2)$ using the result from part (a.)

(5.)(2 pts.) Let $A = \begin{pmatrix} 0.780 & 0.563 \\ 0.913 & 0.659 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1.124 \\ 0.373 \end{pmatrix}$

(a.) Find the condition number of A with respect to the 1-norm.

(b.) A solution to $A\mathbf{x} = \mathbf{b}$ is calculated by first rounding the entries in A to the nearest 0.01 and then row reducing. This calculated solution is $\mathbf{x}_c = (102.4923, -140.750)^T$. Find an upper bound on the relative error with this calculation with respect to the 1-norm.