

## Some Notes on the Final Exam

- (1.) Know how to solve a system of linear equations.
  - (a.) How to form the augmented matrix.
  - (b.) How to row reduce by hand.
  - (c.) How to use MATLAB to row reduce.
  - (d.) Understand the number of solutions that a linear system has.
  - (e.) How to find the general solution.
- (2.) Know what an elementary matrix is.
  - (a.) How does multiplication by an elementary matrix correspond to row reduction.
  - (b.) How to find the inverse of a matrix.
  - (c.) How to use elementary matrices to find a determinant.
- (3.) Know the algebra of matrices
  - (a.) Scalar multiplication, matrix addition, and matrix multiplication.
  - (b.) Transposes of matrices.
- (4.) Understand vector spaces.
  - (a.) Know the primary examples of vector spaces:  $\mathbb{R}^n$ ,  $\mathbb{R}^{m \times n}$ ,  $\mathbb{P}_n$  and  $C[a, b]$ .
  - (b.) What a subspace is and how to determine if a subset is a subspace
  - (c.) How to tell if a set of vectors spans a subspace.
  - (d.) How to tell if a set of vectors is linearly independent.
  - (e.) How to compute and use a Wronskian.
  - (f.) How to find a basis and determine the dimension of a subspace.
  - (g.) Know what the column space, row space and nullspace of a matrix is and how to find a basis for each.
- (5.) Understand inner product spaces.
  - (a.) Know the standard inner products on our primary examples of vector spaces.
  - (b.) How to determine if a function is an inner product.
  - (c.) How to compute the inner products, norms, angles between and orthogonal projections of vectors with a given inner product.
  - (d.) Know what an orthogonal and an orthonormal basis is.
  - (e.) How to compute the orthogonal projection of a vector onto a subspace.
  - (f.) Know the relationship between the column space of a matrix and the nullspace of its transpose.
- (6.) Understand the method of least squares.
  - (a.) Know that the least square approximation is just the orthogonal projection onto a subspace.
  - (b.) Know how to find all the least square solutions to a linear system.
  - (c.) Know how to find the  $n$ th order Fourier series of a function.
- (7.) Understand norms and condition numbers.
  - (a.) How to determine if a given function is a norm.
  - (b.) Know the primary examples of norms on  $\mathbb{R}^n$  and  $\mathbb{R}^{m \times n}$ .
  - (c.) How to compute the norm of a vector or matrix with a given norm.
  - (d.) How to find the condition number of a matrix.
  - (e.) How to find an upper and lower bound of the relative error in some calculation
- (8.) Know the basic MATLAB commands to do the above.