Review 1

(1.) Consider the following system of linear equations:

$$\begin{aligned} 3x_1 - 2x_2 + 4x_3 + 3x_4 - x_5 &= 17 \\ 2x_1 + 4x_3 + x_4 + 2x_5 &= 8 \\ -2x_1 + 2x_2 - 2x_3 - x_4 + 2x_5 &= -10 \\ -2x_1 - 2x_2 - 6x_3 + 2x_4 - 6x_5 &= 0 \end{aligned}$$

(a.) Find the augmented matrix of this system.
(b.) Find the reduced row echelon form of the matrix in (a.).
(c.) Is this system consistent? If so, find all solutions.

(2.) Given the augmented matrix A= $\left(\begin{array}{ccc|c} 1 & -1 & 3 & 1 \\ 3 & 2 & -2 & -2 \\ -2 & 7 & \alpha & \beta \end{array}\right)$, find all values of α and β such that the corresponding linear system has:
(a.) No solutions.
(b.) A unique solution.
(c.) Infinitely many solutions.

(3.) Determine if S is a subspace of V where:
(a.) $V = \mathbb{R}^{2\times 2}$ and S is the set of 2×2 matrices A with det(A)=0.
(b.) $V = \mathbb{R}^{2\times 2}$ and S is the set of 2×2 upper triangular matrices.
(c.) $V = \mathbb{R}^2$ and $S = \{(x_1, x_2)^T \mid |x_1| = |x_2|\}$.
(d.) $V = \mathbb{P}_5$ and S is the set of all polynomials $p(x)$ in V such that $p(1) = 0$.
(e.) $V = C[-1, 1]$ and S is the set of odd functions in V.

(4.) Suppose $A = \begin{pmatrix} 2 & 1 & 3 \\ 0 & 0 & 7 \\ 4 & 0 & 11 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 1 & 3 \\ 0 & -2 & 5 \\ 0 & 0 & 1 \end{pmatrix}$

(a.) Find three elementary matrices $\mathcal{E}_1$, $\mathcal{E}_2$, and $\mathcal{E}_3$ such that $\mathcal{E}_3\mathcal{E}_2\mathcal{E}_1 A = B$.
(b.) Compute det(A) using the results of part (a.). DO NOT USE MATLAB.

(5.) Find an LU-factorization for $A = \begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & 7 \\ -2 & 4 & 1 \end{pmatrix}$. DO NOT USE MATLAB

(6.) True or False. You do not need to explain your answer.

(a.) If A and B are $n \times n$ matrices, then $\det(A + B)$=$\det(A)$+$\det(B)$.
(b.) If A and B are $n \times n$ matrices, then $AB = BA$.
(c.) Suppose $A\mathbf{x} = \mathbf{b_1}$ has a unique solution, then it is possible for $A\mathbf{x} = \mathbf{b_2}$ to have more than one solution.
(d.) An underdetermined system of linear equations is always consistent.
(e.) Every $n \times n$ elementary matrix is invertible.
(f.) If A is an $n \times n$ invertible matrix, then A can be row reduced to I_n.
(g.) If A is an $n \times n$ matrix and α is a scalar, then $\det(\alpha A) = \alpha \det(A)$.

(7.) Prove the following:

(a.) If A is an invertible, then $\det(A^{-1}) = \det(A)^{-1}$.
(b.) If A is an invertible, then $A\mathbf{x} = \mathbf{b}$ has a unique solution for each $\mathbf{b}$.
(c.) If A is an invertible, then $A = \mathcal{E}_1\mathcal{E}_2 \cdots \mathcal{E}_k$ where $\mathcal{E}_1, \mathcal{E}_2, \ldots \mathcal{E}_k$ are some elementary matrices.
(d.) If $A\mathbf{x}_1 = \mathbf{b}$ and $A\mathbf{x}_2 = \mathbf{b}$, then $(\mathbf{x}_1 - \mathbf{x}_2)$ is a solution to $A\mathbf{x} = \mathbf{0}$.