

Project 2

Instructions: This project is worth a total of 10 points. You may use any notes or books that you wish but you must work individually. The only computation aid which you may use is MATLAB, unless otherwise indicated. The primary reference for this project are the notes on Markov processes which can be found at: http://www.math.ohio-state.edu/~husen/teaching/571/markov_1.pdf. Make sure to write clearly and justify your answers.

(1.) (5 pts.) Suppose that one type of atom can be in one of three quantum states which are denoted Q_1 , Q_2 and Q_3 . Given a collection of these atoms, it is found that each measurement perturbs the atoms and each atom has a chance of switching its quantum state. The probabilities that they do are given by:

$$\begin{array}{ll} Q_1 \rightarrow Q_2; P = 0.3975 & Q_1 \rightarrow Q_3; P = 0.1935 \\ Q_2 \rightarrow Q_1; P = 0.3220 & Q_2 \rightarrow Q_3; P = 0.4885 \\ Q_3 \rightarrow Q_1; P = 0.4060 & Q_3 \rightarrow Q_2; P = 0.2665 \end{array}$$

- (a.) Find the transition matrix M of this Markov process.
- (b.) Suppose that initially there are 118 atoms in state Q_1 , 26 in state Q_2 and 47 in state Q_3 . What is the approximate resulting distribution after 7 measurements?
- (c.) Find the steady-state probability vector \mathbf{x}_s of this process.

(2.) (5 pts.) Suppose that one type of atom can exist in a ground state G or be in one of 4 excited states E_1 , E_2 , E_3 and E_4 . Suppose that a collection of these atoms is irradiated by a photon pulse. If an atom is in the ground state, then it has a probability of 0.4389 of being excited to state E_1 , a probability of 0.2599 of being excited to state E_2 and it never transitions to E_3 or E_4 . If it is in state E_1 it has a probability of 0.4709 of dropping to the ground state, a probability of 0.3596 of being excited to state E_2 a probability of 0.0292 of being excited to state E_3 and a probability of 0.0292 of being excited to state E_4 . If it is in state E_2 it has a probability of 0.2305 of dropping to the ground state, a probability of 0.2581 that it transitions to state E_1 and it never transitions to E_3 or E_4 . If it is in state E_3 or E_4 it has a 0.9623 probability of dropping to state E_2 and a 0.0377 probability of remaining in its current state.

- (a.) Find the transition matrix M of this Markov process.
- (b.) Find the steady-state probability vector \mathbf{x}_s of this process.
- (c.) Approximately how many bursts will it take for this process to be within 4 decimal places of accuracy of the steady-state?