Math 572
Winter 2010

Solutions to HW3

5.4

(8a.) $\theta = \cos^{-1}\left(\dfrac{\langle 1, x\rangle}{\|1\|\cdot\|x\|}\right) \Rightarrow \langle 1, x\rangle = \int_0^1 x\,dx = x^2/2\Big|_0^1 = 1/2$

$$\|1\| = \sqrt{\int_0^1 1\,dx} = 1$$

$$\|x\| = \sqrt{\int_0^1 x^2\,dx} = \sqrt{\frac{x^3}{3}\Big|_0^1} = \sqrt{1/3}$$

$$\Rightarrow \boxed{\theta = 1.2780}$$

(8b.) $\boxed{\bar{p} = \dfrac{\langle 1, x\rangle}{\langle x, x\rangle}\, x = \dfrac{1/2}{1/3}\, x = \dfrac{3}{2} x}$

$$\langle 1 - \tfrac{3}{2}x, \tfrac{3}{2}x\rangle = \int_0^1 \frac{3}{2}x - \frac{9}{4}x^2\,dx = \frac{3}{2}\frac{x^2}{2}\Big|_0^1 - \frac{9}{4}\frac{x^3}{3}\Big|_0^1 = 0$$

$\Rightarrow$ $1 - \bar{p}$ and $\bar{p}$ are orthogonal.

(8c.) $\|\bar{p}\| = \sqrt{\int_0^1 (3/2\,x)^2\,dx} = \sqrt{\int_0^1 \frac{9}{4}x^2\,dx} = \sqrt{\frac{3}{4}}$

$\|1\| = 1$

$$\|1 - \bar{p}\| = \sqrt{\int_0^1 1 - 3x + \frac{9}{4}x^2\,dx} = \sqrt{x - \frac{3}{2}x^2 + \frac{3x^3}{4}\Big|_0^1} = \sqrt{\frac{1}{4}}$$

$\|1\|^2 = 1 \qquad \|1 - p\|^2 = \frac{1}{4} \qquad \|\bar{p}\|^2 = \frac{3}{4}$

$$\Rightarrow \|1\|^2 = \|1 - p\|^2 + \|p\|^2$$

$\boxed{5.5}$

(6.) $\langle \bar{u}, \bar{v} \rangle = \langle \bar{u}_1 + 2\bar{u}_2 + 2\bar{u}_3, \bar{u}_1 + 7u_3 \rangle = \langle \bar{u}_1, \bar{u}_1 \rangle + \langle 2u_3, 7u_3 \rangle$

$= 1 + 14 \boxed{= 15}$

$\|\bar{u}\| = \sqrt{\langle \bar{u}, \bar{u} \rangle} = \sqrt{\langle \bar{u}_1, \bar{u}_1 \rangle + \langle 2\bar{u}_2, 2\bar{u}_2 \rangle + \langle 2\bar{u}_3, 2\bar{u}_3 \rangle} = \sqrt{1+4+4} \boxed{= 3}$

$\|\bar{v}\| = \sqrt{\langle \bar{v}, \bar{v} \rangle} = \sqrt{\langle \bar{u}_1, \bar{u}_1 \rangle + \langle 7u_3, 7u_3 \rangle} = \sqrt{50}$

$\Rightarrow \theta = \cos^{-1}\left(\frac{15}{3 \cdot \sqrt{50}}\right) = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \boxed{\frac{\pi}{4}}$

(9a.) $\sin^4 x = \frac{3}{8} - \frac{\cos 2x}{2} + \frac{\cos 4x}{8} = \boxed{\frac{3\sqrt{2}}{8} \cdot \frac{1}{\sqrt{2}} - \frac{1}{2}\cos 2x + \frac{1}{8}\cos 4x}$

(bi.) $\int_{-\pi}^{\pi} \sin^4 x \cos x \, dx = \pi \langle \sin^4 x, \cos x \rangle = \pi \cdot 0 \boxed{= 0}$

(bii) $\int_{-\pi}^{\pi} \sin^4 x \cos 2x \, dx = \pi \langle \sin^4 x, \cos 2x \rangle = \pi\left(-\frac{1}{2}\right) = \boxed{-\frac{\pi}{2}}$

(biii.) $\int_{-\pi}^{\pi} \sin^4 x \cos 3x \, dx = \pi \langle \sin^4 x, \cos 3x \rangle = \pi \cdot 0 \boxed{= 0}$

(biv.) $\int_{-\pi}^{\pi} \sin^4 x \cos 4x \, dx = \pi \langle \sin^4 x, \cos 4x \rangle = \pi \cdot \left(\frac{1}{8}\right) = \boxed{\frac{\pi}{8}}$

$\boxed{5.5}$

(27a) $\langle 1, x\rangle = \int_{-1}^{1} x\,dx = \frac{x^2}{2}\Big|_{-1}^{1} = 0 \Rightarrow$ they are orthogonal

(27b.) $\|1\| = \sqrt{\langle 1,1\rangle} = \sqrt{\int_{-1}^{1} 1\,dx} = \sqrt{x\big|_{-1}^{1}}\ \boxed{= \sqrt{2}}$

$\|x\| = \sqrt{\langle x, x\rangle} = \sqrt{\int_{-1}^{1} x^2\,dx} = \sqrt{\frac{x^3}{3}\Big|_{-1}^{1}}\ \boxed{= \sqrt{2/3}}$

(27c.) $\bar{p} = \bar{p}_1 + \bar{p}_2$ where $\bar{p}_1 = \frac{\langle x^{1/3}, 1\rangle}{\langle 1,1\rangle} 1$ and $\bar{p}_2 = \frac{\langle x^{1/3}, x\rangle}{\langle x,x\rangle} x$

$\Rightarrow$ $\langle x^{1/3}, 1\rangle = \int_{-1}^{1} x^{1/3}\,dx = \frac{x^{4/3}}{4/3}\Big|_{-1}^{1} = 0 \Rightarrow \bar{p}_1 = 0$

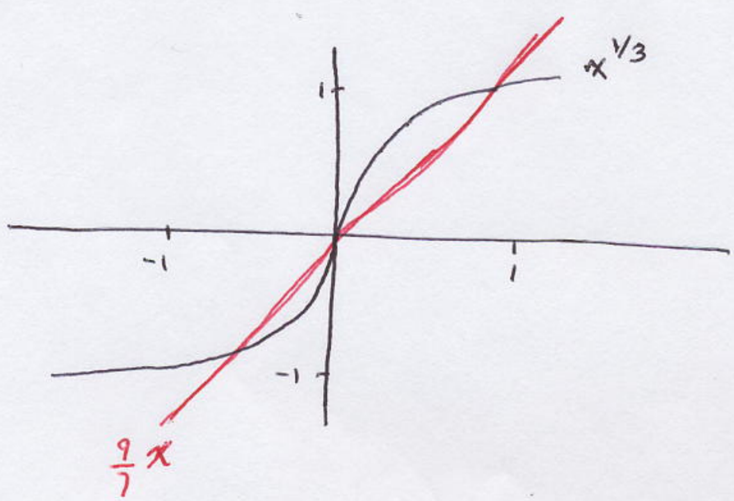
$\langle x^{1/3}, x\rangle = \int_{-1}^{1} x^{4/3}\,dx = 2\int_{0}^{1} x^{4/3}\,dx = 2\frac{x^{7/3}}{7/3}\Big|_{0}^{1} = \frac{6}{7}$

$\Rightarrow p_2 = \frac{6/7}{2/3}x = \frac{9}{7}x$

$\Rightarrow$

$\Rightarrow \boxed{\bar{p} = \frac{9}{7}x}$

(27d.)



$\boxed{5.6}$

(8.) $\bar{y}_1 = \bar{x}_1 = \begin{pmatrix} 4 \\ 2 \\ 2 \\ 1 \end{pmatrix} \Rightarrow \boxed{\bar{z}_1 = \begin{pmatrix} 0.8 \\ 0.4 \\ 0.4 \\ 0.2 \end{pmatrix}}$

$$\bar{y}_2 = \bar{x}_2 - \frac{\langle \bar{x}_2, \bar{y}_1 \rangle}{\langle \bar{y}_1, \bar{y}_1 \rangle} \bar{y}_1 = \begin{pmatrix} 0.4 \\ -0.8 \\ -0.8 \\ 1.6 \end{pmatrix} \Rightarrow \boxed{\bar{z}_2 = \begin{pmatrix} 0.2 \\ -0.4 \\ -0.4 \\ 0.8 \end{pmatrix}}$$

$$y_3 = \bar{x}_3 - \frac{\langle \bar{x}_3, \bar{y}_1 \rangle}{\langle \bar{y}_1, \bar{y}_1 \rangle} \bar{y}_1 - \frac{\langle \bar{x}_3, \bar{y}_2 \rangle}{\langle \bar{y}_2, \bar{y}_2 \rangle} \bar{y}_2 = \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix}$$

$$\Rightarrow \boxed{\bar{z}_3 = \begin{pmatrix} 0 \\ 0.7071 \\ -0.7071 \\ 0 \end{pmatrix}}$$