Math 572
Winter 2010

Exam 1 Review

(1.) Determine if $L : V \to W$ is a linear transformation:
(a.) $L : \mathbb{R}^3 \to \mathbb{R}^2$ and $L(\mathbf{x}) = (x_3 + x_2, x_1 + x_2)^T$.
(b.) $L : \mathbb{P}_3 \to \mathbb{R}^2$ and $L(p(x)) = (p(0), p(1) + 1)^T$.
(c.) $L : C^1[-1, 1] \to \mathbb{P}_2$ and $L(f(x)) = f'(0)x + f(0)$.

(2.) Let $L : \mathbb{R}^3 \to \mathbb{R}^3$ be given by $L(\mathbf{x}) = (x_1 - x_2, x_2 - x_3, x_3 - x_1)^T$. Find a basis for $\ker(L)$ and $L(\mathbb{R}^3)$.

(3.) Determine the matrix representation of $L : V \to W$ with respect to the given bases (for V and W, respectively):
(a.) $L : \mathbb{R}^3 \to \mathbb{R}^2$ and $L(\mathbf{x}) = (2x_1 - x_2, x_2 - 4x_3)^T$ with respect to $[\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3]$ and $[\mathbf{e}_1, \mathbf{e}_2]$.
(b.) $L : \mathbb{P}_3 \to \mathbb{P}_2$ and $L(p(x)) = p(1)p'(x) + p(-1)$ with respect to $[x^2, x, 1]$ and $[x, 1]$.

(4.) Let $L : \mathbb{R}^3 \to \mathbb{R}^2$ be given by $L(\mathbf{x}) = (3x_1 + x_2, x_3 - 2x_2)^T$. Find the matrix representation of L with respect to the bases $[\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3]$ (for $\mathbb{R}^3$) and $[\mathbf{v}_1, \mathbf{v}_2]$ (for $\mathbb{R}^2$) where

$$\mathbf{u}_1 = \begin{pmatrix} 5 \\ 7 \\ 1 \end{pmatrix}, \mathbf{u}_2 = \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix}, \mathbf{u}_3 = \begin{pmatrix} -6 \\ -5 \\ 7 \end{pmatrix} \text{ and } \mathbf{v}_1 = \begin{pmatrix} 2 \\ -2 \end{pmatrix}, \mathbf{v}_2 = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

(5.) Suppose that $L : \mathbb{P}_3 \to \mathbb{P}_3$ and that the matrix representation of L with respect to basis $[x^2, x, 1]$ for $\mathbb{P}_3$ is given by:

$$A = \begin{pmatrix} 2 & -7 & 1 \\ -1 & -3 & 1 \\ 3 & 1 & 0 \end{pmatrix}$$

Find the matrix representation of L with respect to the basis $[3x^2 + 2x + 11, x^2 + 1, -9x^2 - 8x + 7]$.

(6.) Let $S = \text{Span}(1, x, 3x^2 - 1)$ be the subspace of $C[-1, 1]$.
(a.) Verify that $[1, x, 3x^2 - 1]$ is an orthogonal basis for S.
(b.) Find the least squares approximation of $x^{4/3}$ in S.

(7.) Find an orthonormal basis for the subspace S of $\mathbb{R}^4$ spanned by the vectors:

$$\mathbf{x}_1 = \begin{pmatrix} -1 \\ -3 \\ 0 \\ -2 \end{pmatrix}, \mathbf{x}_2 = \begin{pmatrix} -9 \\ -6 \\ 2 \\ 3 \end{pmatrix}, \mathbf{x}_3 = \begin{pmatrix} 2 \\ 1 \\ 0 \\ -2 \end{pmatrix}, \mathbf{x}_4 = \begin{pmatrix} -1 \\ -6 \\ 0 \\ 2 \end{pmatrix}$$

(8.) Find an orthonormal basis for the subspace S of $C[0, 1]$ spanned by the vectors $1, x, x^2$.