EXERCISES TO THE LECTURE OF G. ARZHANTSEVA ON RANDOM GROUPS

E=easy, M=medium, H=hard, O=open and ?="I have never thought about that question, it randomly popped in my head".

Exercise 1 (E). Construct $\Gamma(H)$ such that $[F_2 \colon H] = 6$.

Exercise 2 (E). Show that the composition of finitely many generic properties is generic.

Exercise 3 (E). Show that the following presentation of the fundamental group of an oriented compact surface of genus $g > 1$ satisfies the small cancellation condition $C'(\lambda)$ (give an estimate on λ)

$$S_g = \langle a_1, b_1, \dots, a_g, b_g \mid a_1 b_1 a_1^{-1} b_1^{-1} \cdots a_g b_g a_g^{-1} b_g^{-1} = 1 \rangle.$$

Exercise 4 (E). Show that the property "there is no proper powers among reltors" (that is, $\forall r \in R$ we have $r \neq w^d$ for $d \geqslant 2$ and a non-empty word w) is generic.

This, together with the genericity of $C'(\lambda)$-condition, implies that a generic f.p. group is torsion-free.

Exercise 5 (M). Prove that $H \leqslant F_m$ is of finite index $\iff$ $\forall v$ is of max. degree in a reduced $\Gamma(H)$. What about $H \leqslant G$ for a non-free group G?

Exercise 6 (H). Prove that a generic finitely presented group has a free non-abelian subgroup.

Hence, a generic finitely presented group is of exponential growth and, moreover, non-amenable.

Indication: use $C'(1/6)$-condition and assume that relators are long enough.

Exercise 7 (H). In the density model of a random group G, show that $d > \frac{1}{2} \Longrightarrow G$ is $\{e\}$ or $\mathbb{Z}/2\mathbb{Z}$.

Exercise 8 (O). Prove that every subgroup of infinite index is free in a generic finitely presented group.

It is known for subgroups with a given prescribed number of generators (Arzh.'98).

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