Exercises on Thompson's Groups

Let L_n be the following simplicial complex. Consider a row of $n+2$ points. A vertex in L_n is an edge connecting two neighboring points, and a collection of such edges forms a simplex if they are pairwise disjoint.

Exercise 1 (E). Show that

$$L_{n+2} = L_{n+1} \cup_{L_n} \operatorname{Cone}(L_n)\,.$$

Exercise 2 (M). Show the following homotopy equivalences:

$$L_n \simeq \begin{cases} * & \text{for } n = 3m \\ \mathbb{S}^m & \text{for } n = 3m+1 \text{ or } n = 3m+2 \end{cases}$$

Exercise 3 (E). Show that F is torsion free.

Exercise 4 (M). Show that F has infinite cohomological dimension.

Exercise 5 (M). Show that F is not the fundamental group of a 3-manifold. (Hint: you might find the previous exercise useful.)

Exercise 6 (H). Solve the conjugacy problem in F.

Exercise 7 (O). Solve arbitrary systems of equations in F.

Exercise 8 (O (H?)). Decide whether F is amenable.

Exercise 9 (M). Show that

$$F = \left\langle a, b \,\middle|\, b^{aa} = b^{ab},\ b^{abb} = b^{abb^a} \right\rangle .$$

Exercise 10 (H). Show that V is simple.

Exercise 11 (E). Show that F is not residually finite.

Exercise 12 (E). Show that F has free abelian subgroups of arbitrary high rank.

Exercise 13 (M). Show that F is not residually linear.

Exercise 14 (M). Show that x_0^{-1} and x_1 generate a free monoid inside F. Deduce that F has exponential growth.

Exercise 15 (M). Show that V contains non-abelian free subgroups.

Exercise 16 (?). Give an explicit contraction of $\tilde{X}$ inspired by the pictures: