

MAY 15, 2009

(1, E) Show that in the Grigorchuk group  $\Gamma$

- (1)  $\langle a, d \rangle$  is a dihedral group of order 8;
- (2)  $\langle a, c \rangle$  is a dihedral group of order 16;
- (3)  $\langle a, b \rangle$  is a dihedral group of order 32.

(2, O) Does there exist a finitely generated group of superpolynomial growth slower than  $\exp(\sqrt{n})$ ?

(3, O) Is it true that in the Grigorchuk group  $\Gamma$  with the metric given by the generators  $\{a, b, c, d\}$  the set of balls  $B_{id}(n)$  is a Følner sequence?

(4, O) Does there exist a finitely presented group of intermediate growth?

(5, E)

- (1) Prove that the growth function  $\alpha : \mathbb{N} \rightarrow \mathbb{N}$  given by  $\alpha(n) = |B_{id}(n)|$  is monotone increasing.
- (2) Prove that  $\alpha(n + m) \leq \alpha(n)\alpha(m)$ .
- (3) Prove that any finitely generated group has at most exponential growth.

(6, M++) Show that the word problem in  $\Gamma$  is solvable in  $O(n \log n)$  time.

(7, E–M) Draw the Schreier graphs of the Basilica group and of the 3-pegs Hanoi game group for the small values of  $n$ . Figure out combinatorial rules which allow to construct the next Schreir graph from the previous one.