XPPAUT Tutorial for Math 865L

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XPPAUT is a software with a graphic interface for solving and analysis ODEs. Here we give the instruction to the basic command.

1. Connection to math account: ssh -Y account_name@compute.math.ohio-state.edu It will ask for password for permission.

2. To start XPPAUT: [account_name@compute ~]$ xppaut Math865L_example1.ode

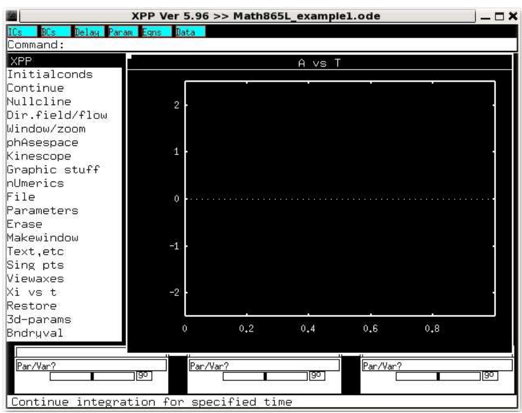

Figure 1: xppaut

Math865L_example1.ode is a script for XPPAUT to solve the example (3.3)

$$\frac{d}{dt}\begin{bmatrix}[A]\\ [B]\\ [C]\end{bmatrix} = \begin{bmatrix}-1 & 1\\ -2 & 2\\ 1 & -1\end{bmatrix}\begin{bmatrix}v_1\\ v_2\end{bmatrix}$$

where $v_1 = k_1[A][B]^2$ and $v_2 = k_2[C]$.

Algorithm 1 Math865L_example1.ode

```
# Math865L_example1.ode (XPPAUT file) written by Chiu-Yen Kao
# This is an XPP file to numerically integrate the equations in p.19 # in the
first handout : Ch3 Mathematical and computational modeling tools
# SYSTEM OF EQUATIONS
dA/dt = -k1*A*B^2+k2*C
dB/dt = -2*k1*A*B^2+2*k2*C
dC/dt = k1*A*B^2-k2*C
# PARAMETERS
parameter k1=0.5
parameter k2=0.3
# INITIAL CONDITIONS
init A=2.0,B=1.5, C=1.0
# CHANGES FROM XPP'S DEFAULT VALUES
@ TOTAL=1.0,DT=0.01,XLO=0.0,XHI=1.0,YLO=-2.5,YHI=2.5
done
```

3. To quit XPPAUT, click on File and then Quit

4. To solve with parameters listed in Math865L_example.ode by XPPAUT, click on Initialconds, and then Go (IG on the keyboard). Notice that only A is plotted!!

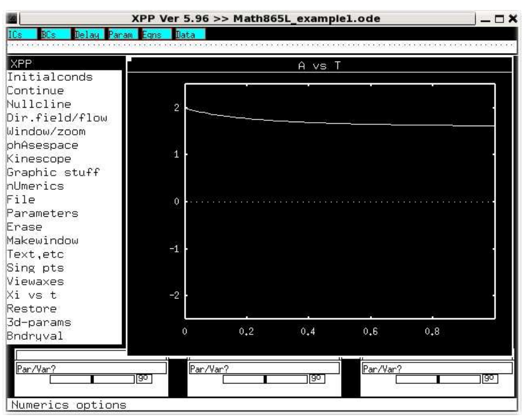

Figure 2: A v.s. t for $0 \leq t \leq 1$

5. To continue the solution from where it ended up by clicking on Initialconds and then Last. It will overlapped with previous plot!! To clear the graphing window, click on Erase.

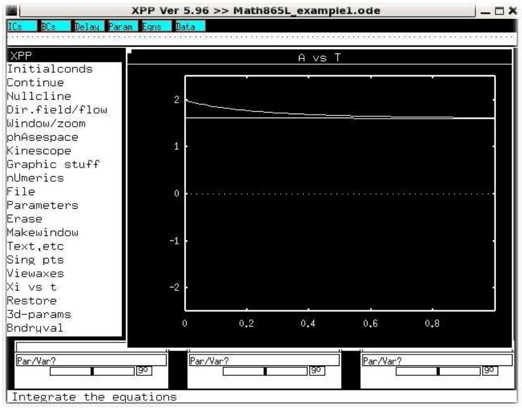

Figure 3: A v.s. t for $0 \leq t \leq 1$ and $1 \leq t \leq 2$

6. To add a graph, click on Graphic stuff-Add curve. Fill in the popup window with the appropriate information (replace A by B on the y-axis; choose color=3 instead of 0).

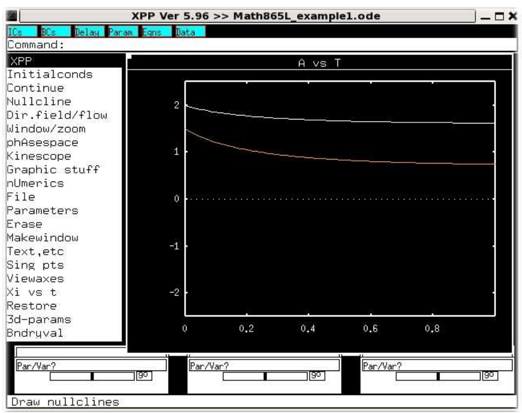

Figure 4: A and B v.s. t for $0 \leq t \leq 1$

7. To plot A in a separate window, click Makewindow- Create (MC on the keyboard). You should get a new window with a plot of A(if you don't see the trajectory, click Restore). To change to a plot of C, click Xi vs t, and fill in the information asked for in the top bar on the main window (replace A with C). The axes will be scaled automatically. To get different scales, experiment with the Window/zoom command and the Viewaxes-2D command.

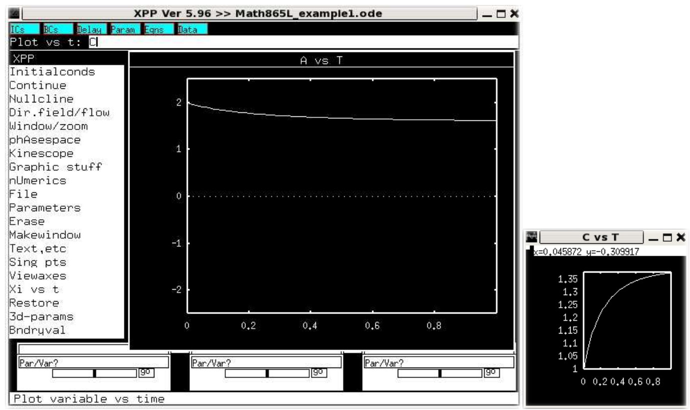

Figure 5: A and C v.s. t for $0 \leq t \leq 1$

8. How to switch between different windows? The active window is the one that has a tiny white square in the top-left corner of the graphing area. You can make a non-active window active by clicking on its graphing area.

9. Phase-plane analysis: Use the Viewaxes-2D command, and fill in the popup window with the appropriate information. Specify the A variable on the x-axis, and the B variable on the y-axis. Also you can adjust the size here.

Figure 6: Viewaxes-2D command, phase plan A and B

10 Click Data on the top to see the data set.

Now Let's go back to the six possible phase portraits listed in Figure 3.3 in the handout. We consider

$$\frac{d}{dt}X = \frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = MX$$

where M is

$$(a) M = \begin{bmatrix} 5 & -1 \\ 3 & 1 \end{bmatrix} \quad \text{node,} \quad \text{unstable}$$
$$(b) M = \begin{bmatrix} -1 & -1 \\ 0 & -0.25 \end{bmatrix} \quad \text{node,} \quad \text{asymptoticallystable}$$
$$(c) M = \begin{bmatrix} 3 & -2 \\ 2 & -2 \end{bmatrix} \quad \text{saddle point,} \quad \text{unstable}$$
$$(d) M = \begin{bmatrix} 2 & -5/2 \\ 9/5 & -1 \end{bmatrix} \quad \text{sprial point,} \quad \text{unstable}$$
$$(e) M = \begin{bmatrix} 1 & -5 \\ 1 & -3 \end{bmatrix} \quad \text{sprial point,} \quad \text{asymptoticallystable}$$
$$(f) M = \begin{bmatrix} 1 & 2 \\ -5 & -1 \end{bmatrix} \quad \text{center} \quad \text{stable}$$

Algorithm 2 Math865L_example2.ode

```
# Math865L_example2.ode (XPPAUT file) written by Chiu-Yen Kao
# This is an XPP file to numerically integrate the equations in p.25 # in the first handout : Ch3 Mathematical and computational modeling tools
# SYSTEM OF EQUATIONS
dx1/dt = m11*x1+m12*x2 dx2/dt = m21*x1+m22*x2
# PARAMETERS
parameter m11=-1 parameter m12=-1 parameter m21=0 parameter m22=-0.25
# INITIAL CONDITIONS
init x1=1.0,x2=1.0
# CHANGES FROM XPP'S DEFAULT VALUES
@ TOTAL=20.0,DT=0.01,XLO=0.0,XHI=20.0,YLO=-2.5,YHI=2.5
done
```

11 To add the nullclines, click Nullcline-New.

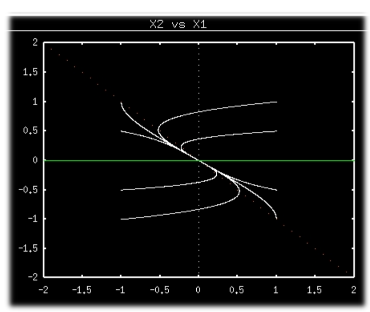

Figure 7: possible Phase Portraits (b)

HW1: Consider the system of 2 nonlinear equations

$$\begin{aligned} \frac{dx_1}{dt} &= f_1(x_1, x_2) \\ \frac{dx_2}{dt} &= f_2(x_1, x_2) \end{aligned}$$

Choose f_1 and f_2 to generate similar phase protraits in Figure 3.4 (p.26) and Figure 3.5 (p.3.5). Write your choice and solve the system by XPPAUT. Study the stability of the zeros of f_1 and f_2.

HW2: Explicit Runge-Kutta methods (ERK) can be written as

$$y_{n+1} = y_n + h \sum_{j=1}^{\nu} b_j f(t_n + c_j h, \xi_j), \quad n = 0, 1, 2, ...$$

where $\xi_j = y(t_n + c_j h)$ and

$$\begin{aligned} \xi_1 &= y_n \\ \xi_2 &= y_n + h a_{2,1} f(t_n, \xi_1) \\ \xi_3 &= y_n + h a_{3,1} f(t_n, \xi_1) + h a_{3,2} f(t_n + c_2 h, \xi_2) \\ &\vdots \\ \xi_\nu &= y_n + h \textstyle\sum_{i=1}^{\nu-1} a_{\nu,i} f(t_n + c_i h, \xi_i) \\ y_{n+1} &= y_n + h \textstyle\sum_{j=1}^{\nu} b_j f(t_n + c_j h, \xi_j) \end{aligned}$$

The matrix is $A = (a_{j,i})_{j,i=1,2,\ldots,\nu}$ is called the RK matrix, while

$$b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_\nu \end{bmatrix} \quad \text{and} \quad c = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_\nu \end{bmatrix}$$

are the RK weights and RK nodes respectively. The two-stage ERK with at least order $p \geq 2$ need to satisfies

$$b_1 + b_2 = 1, \quad b_2 c_2 = \frac{1}{2}, \quad a_{2,1} = c_2.$$

What are the conditions for third order, three-stage ERK method?