XPPAUT Tutorial II for Math 865L

Chiu-Yen Kao

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Last time we show the phase protraint of

$$\frac{d}{dt}X = \frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = MX$$

where M is

$$(a)M = \begin{bmatrix} -1 & -1 \\ 0 & -0.25 \end{bmatrix} \text{node,asymptoticallystable.}$$

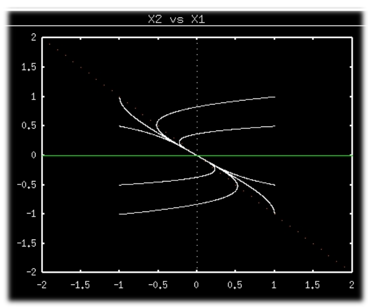

Figure 1: (b)Phase Portraits

Today, we will talk a little bit more about xppaut in phase protraint and bifurcation.

1. Click the 'Erase' command to get a clear graphing area.
2. Use 'Initialconds' and then 'Mouse' command.
3. To find the steady states and the associated eigenvalues, click on 'Sing pts' and then 'Go'. The eigenvalues will be printed in the X-window. In this case, they are $\lambda = -1, -0.25$. If there are several steady states, you can use 'Sing pts' and then 'Mouse' command to get a particular steady state.
4. To plot the direction field, click on 'Dir.field/flow'. Change Grid from the default value 10 to 5.

Algorithm 1 Math865L_example3.ode

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# Math865L_example3.ode (XPPAUT file) written by Chiu-Yen Kao
# This is an XPP file to numerically integrate the equations in p.27 # in the first handout : Ch3 Mathematical and computational modeling tools
# PARAMETERS
param c0=-1 param c1=0, c2=1, c3=0
# INITIAL CONDITIONS
init x=0.5
# SYSTEM OF EQUATIONS
dx/dt = c0+c1*x+c2*x^2+c3*x^3
# CHANGES FROM XPP'S DEFAULT VALUES
@ TOTAL=5.0,DT=0.01,XLO=0.0,XHI=5.0,YLO=-2.5,YHI=2.5
@ dsmin=0.00001, dsmax=0.05, parmin=-1, parmax=1
@ autoxmin=-1, autoxmax=1, autoymin=-1, autoymax=1
done
```

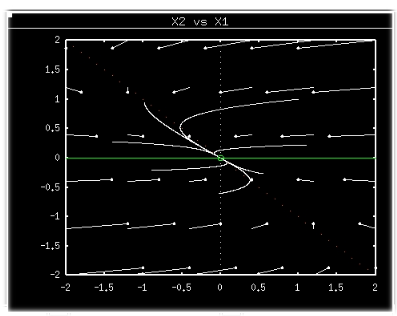

Figure 2: direction field of (b)

Consider a system of ODEs that depends on a parameter p (ref to the handout p. 27)

$$\frac{dx}{dt} = f(x,p) = c_0 + c_1 x + c_2 x^2 + c_3 x^3.$$

We know that

$$\begin{array}{cc} (a) f(x,p) = p + x^2 & \text{saddle-point bifurcatiom} \\ (b) f(x,p) = px - x^2 & \text{transcritical bifurcation} \\ (c) f(x,p) = px - x^3 & \text{pitchfork bifurcation} \end{array}.$$

In the code Math865L_example3.ode, we demonstrate the case

$$c_0 = -1,\ c_1 = 0,\ c_2 = 1,\ c_3 = 0.$$

If we solve this equation with several different initial conditions $x_0 = -1.5, -1, -0.5, 0, 0.5, 1, 1.5$, we can see $x = -1$ is asymptotically stable and $x = 1$ is unstable.

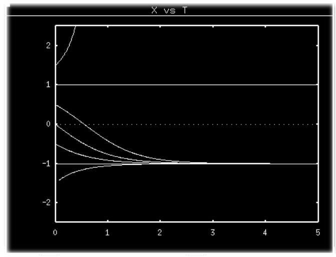

Figure 3: Auto mode

In general,

$$\frac{dx}{dt} = p + x^2$$

have two zeros $x = \pm\sqrt{p}$.

5. To draw bifurcation diagram, you need to start from at a stable attractor, either a stable steady state or a stable limit cycle. Here we demonstrate the case with stable steady state. Use 'Initialconds-Last' command a few times to ensure that your solution has converged to the stable steady state that exists when $c_0 = -1$. Start the AUTO portion of XPPAUT by clicking File-Auto. You should see a new Auto window, with an additional set of menu commands. I've already set all the internal control parameters in the fhn.ode, so you're set to go. Click the Run-Steady state command. You should see the curve of steady states. Thick portions of the curve represent stable steady states; thin portions represent unstable steady states. (You need to start the AUTO portion of XPPAUT at a stable attractor, either a stable steady state or a stable limit cycle.)

6. Click Grab, and press the Tab button your keyboard repeatedly to cycle through important points on the curve of steady states and you can the changing information at the bottom of the Auto window.

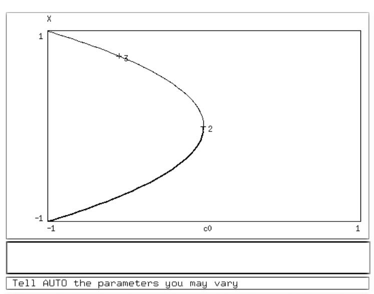

Figure 4: Auto mode

Algorithm 2 Math865L_example4.ode

Math865L_example4.ode (XPPAUT file) written by Chiu-Yen Kao
This is an XPP file to numerically integrate the equations in p.29 # in the first handout : Ch3 Mathematical and computational modeling tools
PARAMETERS
param pp= -0.25 param a=0.25, eps=0.05
INITIAL CONDITIONS
init x=0.3, y=0.3
SYSTEM OF EQUATIONS
x' = 2*x * (1-x) * (x-a) - y + pp y' = eps * (x - y)
CHANGES FROM XPP'S DEFAULT VALUES
@ total=100,DT=0.01,XLO=0.0,XHI=10.0,YLO=-0.4,YHI=1 @ dsmin=0.00001, dsmax=0.05, parmin=-0.5, parmax=1 @ autoxmin=-0.25, autoxmax=1, autoymin=-0.5, autoymax=1.5
done

Now we consider the following system with two time scales

$$\begin{aligned} \frac{dx}{dt} &= f(x,y) \\ \frac{dy}{dt} &= \varepsilon g(x,y) \end{aligned}$$

where $f(x,y) = 2x(1-x)(x-a) - y + p$ and $g(x,y) = x - y$. With different parameters p, we can either have a stable steady state or a stable limit cycle.

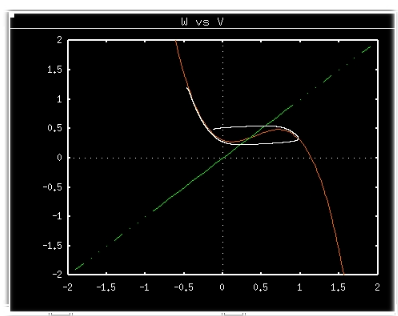

Figure 5: Auto mode

7. Use the Grab command to grab the first first (left-most) Hopf bifurcation (remember to press the Return button to exit the Grab command). Click Run-Periodic to compute the curve of periodics. For each parameter value, you will see a pair of solid dots; the top one represents the maximum value of v during the limit cycle, and the bottom one represents the minimum value.

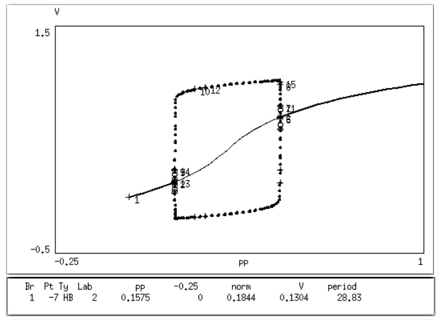

Figure 6: Hopf bifurcation

HW: Generate Figure 3.6 (b) and 3.7(a)(b) by using xppaut.