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Title of Talk: Triangle inequalities on the 2-sphere

Abstract:
The triangle inequalities in the Euclidean plane R^2 are a well-known part of analysis. If A, B and C are the vertices of a geodesic triangle in R^2, and d(X,Y) denotes the distance from X to Y, then d(A,C) <= d(A,B) + d(B,C). This inequality is a necessary and sufficient condition for existence of a triangle in the plane with the given side lengths. Here we investigate abstract triangles embedded in the 2-sphere S^2 with edges are along geodesic arcs, where triangle sides are allowed to wrap around the sphere. We thus allow angles greater than 2*Pi.

These abstract triangles appear in the study of topology and metrics, in locally spherical structures with three conical singularities. Preliminary results show the solution space for these abstract triangles is connected, but not convex and not star-shaped. The inequalities can be derived by using dual triangles with side lengths satisfying the regular inequalities a<=b+c, b<=a+c, c<=a+b, with the additional condition that a+b+c<2*Pi. The solution space is a union of tetrahedrons, each in a quadrants of side length Pi, and the regions in different quadrants are related by composition of reflections.