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Title of Poster Presentation: Bifurcations of the Henon Map: Routes to Chaos

Abstract:
Many simple physical systems exhibit chaotic behavior. A simplification of a weather prediction model due to Lorenz is the Henon map $f(x,y)=(1-ax^2+y, bx)$. This mapping has been devised by the theoretical astronomer Michael Henon to illuminate the microstructure of chaotic attractors and is the subject of intensive research. The importance of the characteristics of the Henon map is that it leads to a better understanding of chaotic behavior.

Our methodology was that we investigated the existence and transformation of the chaotic attractor of the Henon mapping $f(x,y)=(1-ax^2+y, bx)$, as $0<a<2$ and $|b|<1$, with numerical methods.

The classical parameter values are $a=1.4$ and $b=0.3$. For these values, the mapping is contracting the area and has a trapping region, so it exhibits an attractor. However, for all values $|b|<1$, and for a wide range of values of a, the mapping is still contracting the area, and still has a trapping region. The global dependence on (a,b) is largely unexplored in existing literature. We investigated the whole set of parameter values (a,b) that generate chaotic attractors. Our results are shown through bifurcation diagrams.

We developed computer programs using Maple software to explore the dependence of dynamics on the parameters. We introduced a new exploration tool, which is a Mandelbrot-type set of parameter values (a,b) that generate chaotic attractors. This parameter-set has a fairly regular boundary, except for some portions of it, which are fractals. Various regions of this parameter set yield various classes of attractors. We also developed three-dimensional bifurcation diagrams that show the dependence of the Henon attractor on both parameter values, which allow a better understanding of double crises and their loci.