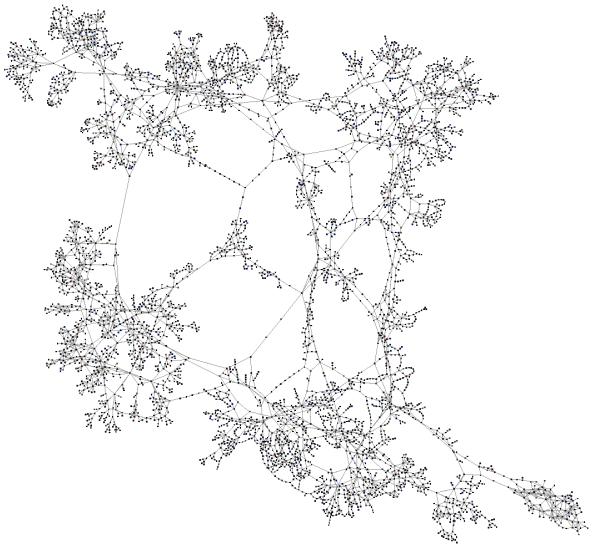
Graphs

A **graph** is a *structure that defines pairwise relationships* within a set to objects. The objects are the vertices, and the pairwise relationships are the edges: *X* is related to *Y* if and only if *XY* is an edge.

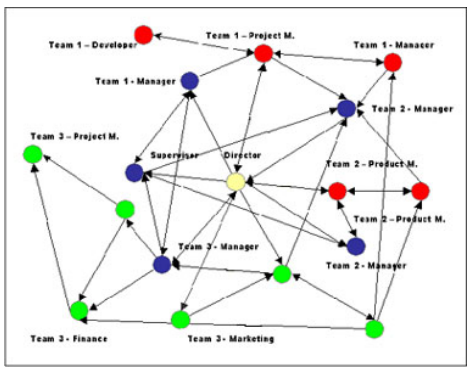
A **graph** is a picture consisting of:

- **Vertices- dots**
- **Edges- lines**
  The edges do not have to be straight lines. But they have to connect two vertices.
- **Loop- an edge connecting a vertex back with itself**

Graph of part of the U.S. Electrical Grid



Organizational Chart for Project



Does this look familiar?

Real-World Graph

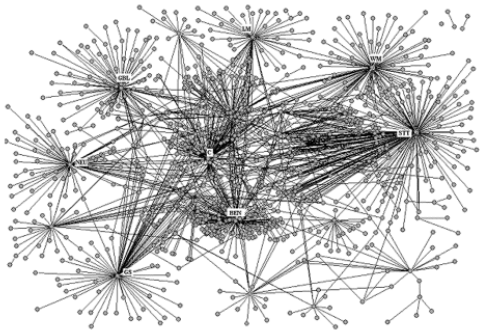
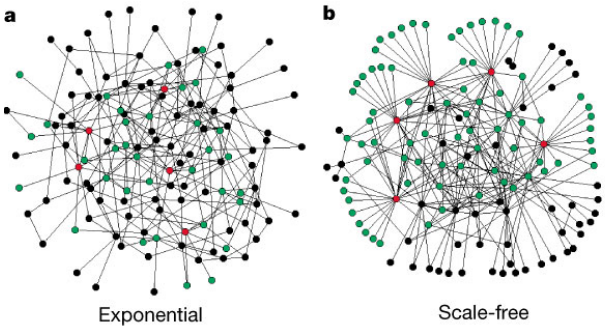


Figure 1. The graph of ownership for stocks traded in 2001 on the New York Stock Exchange.

Nodes = Companies
Edge = portfolios own each others stock

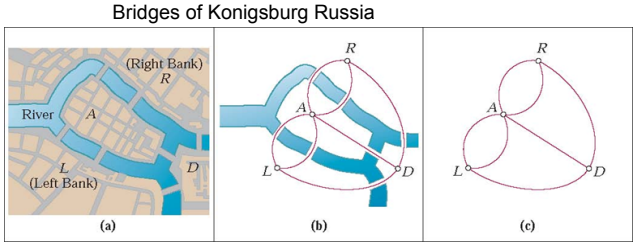
Two Types of Random Graphs



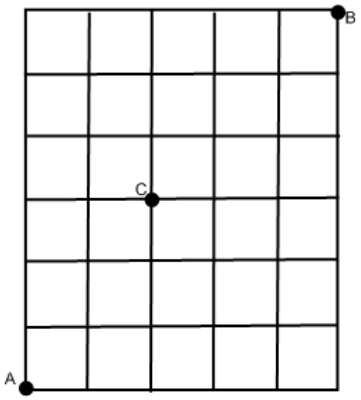
Mathematicians have found to produce random graphs that have similar structures to many real-world graphs. These allow us to analyze the structure to find ways to solve real-world problems.

London Metro

Bridges of Konigsburg Russia



The following graph represents roads on a city grid. How many different routes are there from A to B (Only going North or East)?



Neighborhood South of New York's Central Park

What do mathematicians look for in graphs?

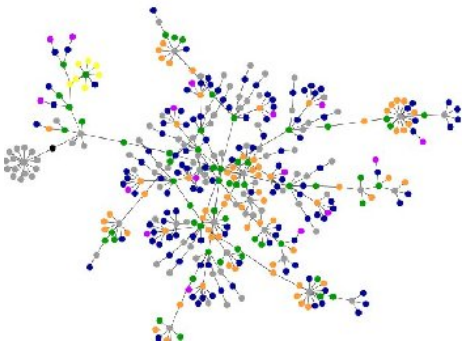
How can you get from one place to another?

How hard is it to send a message from one node to another? to all the others?

How does information (a rumour, or a disease, or a fad) spread across a network?

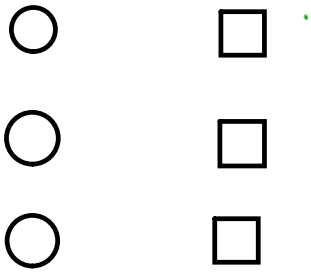
Are some nodes "more important" than others?

Network of "friendships"

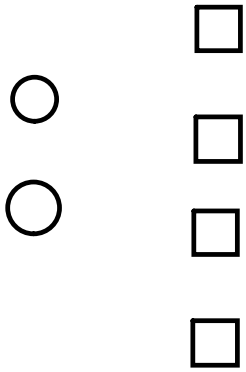


Can you draw the graph without "crossing edges"?

Draw the cables/pipes between three houses (on right) and gas, electric and water companies on the left. Try not to let the pipes cross.



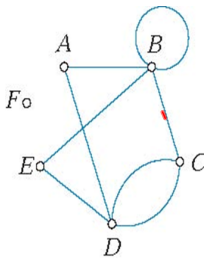
Do the same for these four houses and two utilities.



Real world application: Design computer chips. Need to connect many input sources with many output sources around the edge of a chip. Each Crossing adds cost as well as heat to the chip. You would like as few crossings as possible. How to design it?

Notation for graphs

This graph has six vertices A, B, C, D, E, and F and eight edges. The edges can be described by giving the two vertices that are connected by the edge.



List all the edges: (as a pair of vertices)

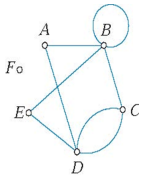
Definitions and Terminology

Two vertices are said to be **adjacent** if

The **degree of a vertex** is

A **path** is a sequence of vertices with the property that each vertex in the sequence is *adjacent* to the next one. The key requirement in a path is that an edge can be part of a path only once.

A **circuit** is...



Attachments

- Web Pages as Graphs
- Euler Circuit
- TheHousesAndUtilitiesCrossingProblem.nbp