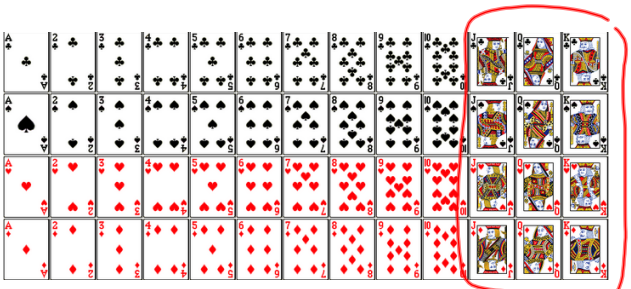
1. Now consider a deck of cards. You are dealt three random cards. How many ways can it happen that:

a. (3 points) They are all face cards.(Jack, Queen or King)

12 face cards $_{12}C_3$ ways to pick 3 cards out of 12

\

b. ~~(3 points) They are all the same suit.~~



$Pr(3 \text{ face cards}) = \frac{\#\text{ways 3 face cards}}{\#\text{ways 3 cards of any kind}} = \frac{{}_{12}C_3}{{}_{52}C_3} = \frac{\left(\frac{12 \cdot 11 \cdot 10}{3 \cdot 2 \cdot 1}\right)}{\left(\frac{52 \cdot 51 \cdot 50}{3 \cdot 2 \cdot 1}\right)} = \frac{12 \cdot 11 \cdot 10}{52 \cdot 51 \cdot 50}$

1st card, 2nd card, 3rd card

3. Suppose you are dealt three cards from a standard deck of 52. Find the probability that:

a. (3 points) They are three of a kind.

Pick rank A-2-K. Pick 3 of 4 cards of that rank.

$$Pr(3 \text{ of a kind}) = \frac{\#3 \text{ of kind}}{\#3 \text{ cards}} = \frac{13 \cdot {}_4C_3}{{}_{52}C_3}$$

b. (3 points) They are all different suits and different values.

1. Suppose you flip 10 coins.

a. (4 points) Find the probability that you get 3 or fewer tails.

0 tails 1 Tail 2 Tails 3 tails

$$Pr(\leq 3 \text{ tails}) = \frac{\#\leq 3 \text{ tails}}{\#\text{possible}} = \frac{1 + 10 + {}_{10}C_2 + {}_{10}C_3}{2^{10}}$$

b. (4 points) Find the probability that you get at least two heads in a row or at least two tails in a row. (Hint: First, find the probability that that does not happen.)

all but 2 possibilities

HTHTHT....

or

THTHTH....

$$Pr(\geq 2 \text{ in a row}) = \frac{1022}{1024}$$

1. Suppose you are apportioning prize to the girl scout troop based on the Girl Scout cookie sales of each girl. You have 13 prizes to hand out to the 5 girls in the troop. Their names are Abby, Beverly, Cindy, Donna and Eve. Based on their current cook sales each girl will get the following numbers of prizes. Abby gets 5, Beverly gets 3, Cindy gets 2, Donna gets 2, and Eve gets 1.

a. (2 points) Describe a situation that would be an example of the **Alabama paradox** in apportioning the prizes.

more prizes given out, someone gets fewer.

a. (2 points) Describe a situation that would be an example of the **new states paradox**.

New girl scout, w/ more prizes reapportion and someone loses a prize.

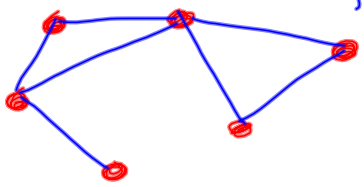
a. (2 points) Describe a situation that would be an example of the **populations paradox**.

One girl sells some more cookies and loses a prize.

Graphs

A **graph** is a *structure that defines pairwise relationships* within a set to objects. The objects are the vertices, and the pairwise relationships are the edges: *X* is related to *Y* if and only if *XY* is an edge.

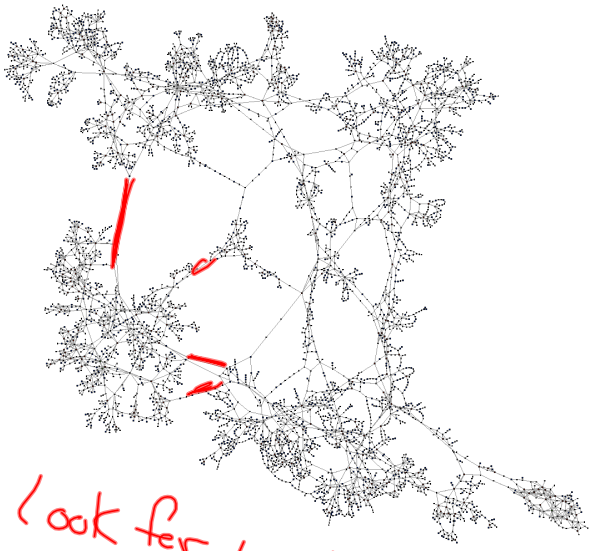
vertex → student
edge → in a class together



A **graph** is a picture consisting of:

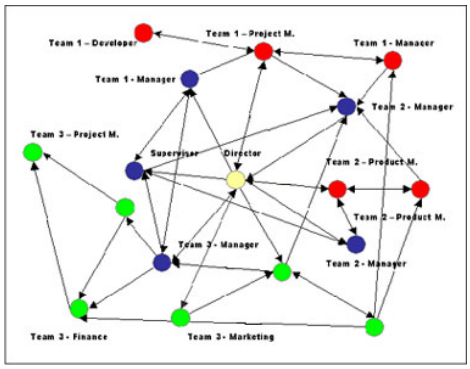
- **Vertices- dots**
- **Edges- lines**
  The edges do not have to be straight lines. But they have to connect two vertices.
- **Loop- an edge connecting a vertex back with itself**

Graph of part of the U.S. Electrical Grid



look for weakness in power grid

Organizational Chart for Project



Does this look familiar?

Facebook

Real-World Graph

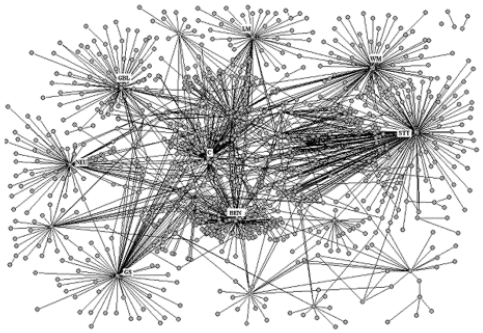
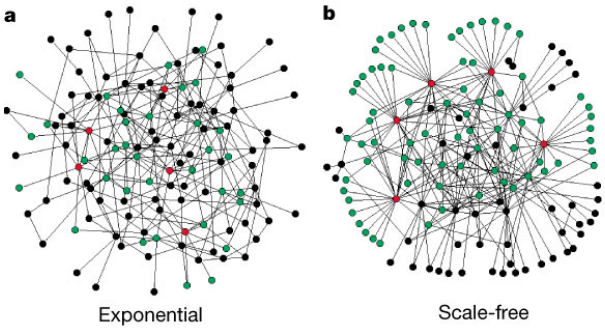


Figure 1. The graph of ownership for stocks traded in 2001 on the New York Stock Exchange.

Nodes = Companies
Edge = portfolios own each others stock

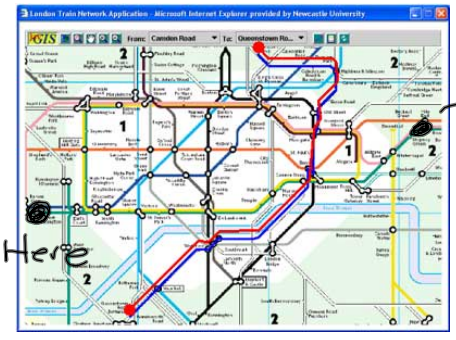
Some companies are more "central" to network

Two Types of Random Graphs



Mathematicians have found to produce random graphs that have similar structures to many real-world graphs. These allow us to analyze the structure to find ways to solve real-world problems.

London Metro

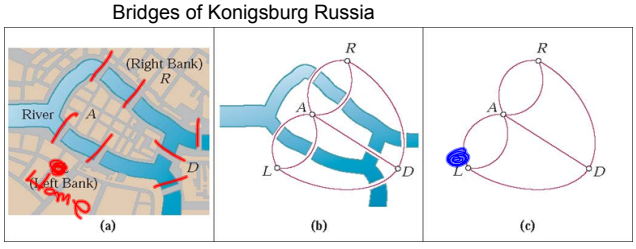


Here

There

How to go from here to there?

Bridges of Konigsburg Russia



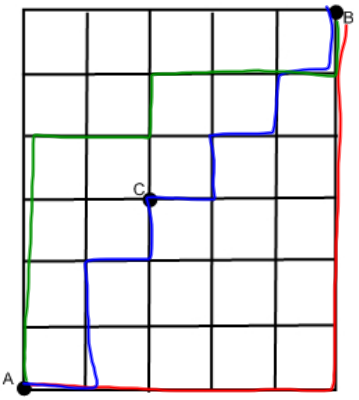
"Oil-er"

Euler
First one to use graphs to look at transportation

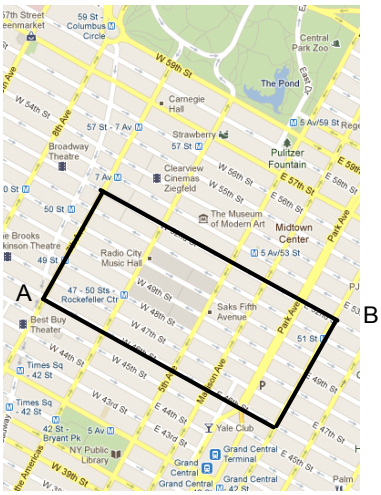
Cross each bridge exactly once and end up at home

Cross every edge exactly 1 time and up back at "C"

The following graph represents roads on a city grid. How many different routes are there from A to B (Only going North or East)?



Neighborhood South of New York's Central Park



$_{11}C_5$ ways to go from A to B.

11 blocks to walk, chose 5 to go east (the other 6 are north)

What do mathematicians look for in graphs?

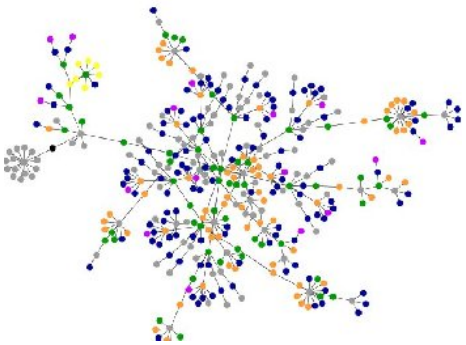
How can you get from one place to another?

How hard is it to send a message from one node to another? to all the others?

How does information (a rumour, or a disease, or a fad) spread across a network?

Are some nodes "more important" than others?

Network of "friendships"



Attachments

- Web Pages as Graphs
- Euler Circuit
- TheHousesAndUtilitiesCrossingProblem.nbp