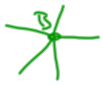
Definitions and Terminology

Two vertices are said to be **adjacent** if

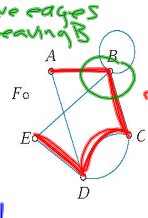
The **degree of a vertex** is how many edges are leaving a node

deg(A) = 2 deg(B) = 5 Five edges leaving B

deg(C) = 3



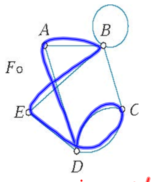
A **path** is a sequence of vertices with the property that each vertex in the sequence is *adjacent* to the next one. The key requirement in a path is that an edge can be part of a path only once. No repeated edge

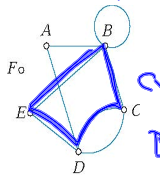


path

A **circuit** is... path that starts and ends at same spot.

A cycle is a circuit with no repeated vertex





cycle

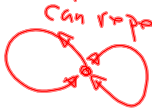
B-C-D-E-B

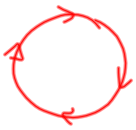
circuit

D-C-D-A-B-E-D

No repeated path but can repeat vertex

cycle





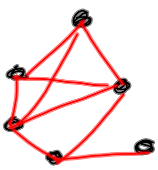
A graph is **connected**, if

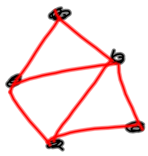
There is a path between any two vertices.

Nodes are students

Edge if in class together

A graph that is not connected is said to be **disconnected**.

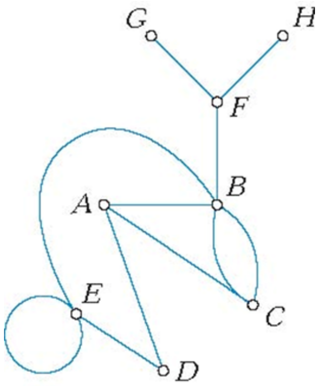




A disconnected graph is made up of separate **components**.

Separate parts of a network.

Sometimes in a connected graph there is an edge such that if we were to erase it, the graph would become disconnected—such an edge is called a **bridge**.

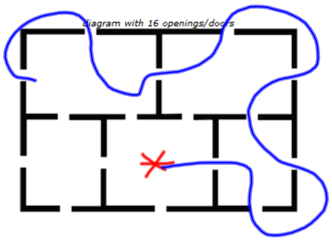


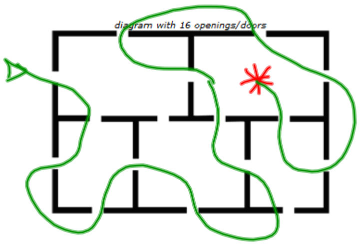
What edges are the bridges in this graph?

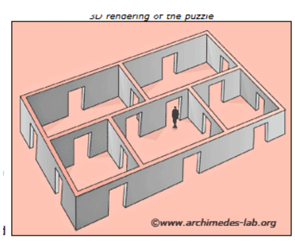
edge(F,B) is a bridge

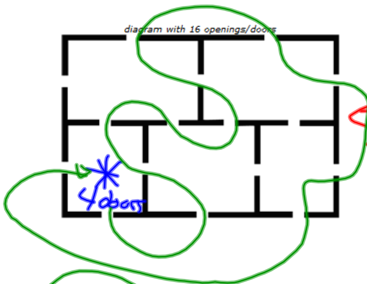
edge (F,G)
(F,H) also bridges

5-Room Puzzle: Is there a way to visit every ~~room~~ door in this house without using the same door twice?



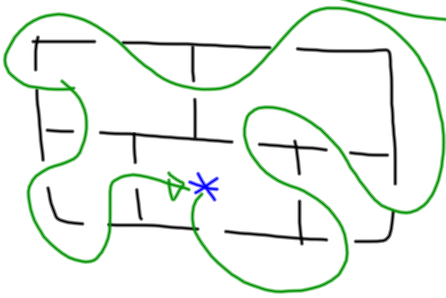






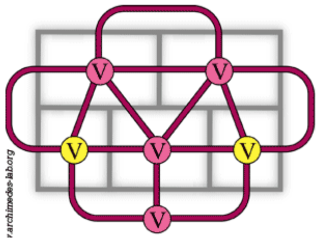
← Impossible

Different Map



← Works
Why?

How is this a graph?



vertices = rooms

edges = if there is a door between two rooms

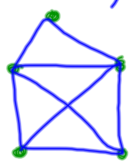
The question becomes....Can you visit every vertex without...

Can you visit every edge exactly once and return to start?

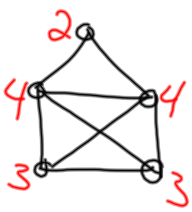
Is there a circuit that goes on every edge exactly one time?

Euler Circuit: A circuit in a graph that crosses every edge exactly once and ends up where it started.

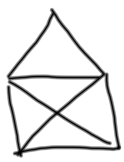
Euler Path: A path that crosses every edge exactly once (doesn't end where it started.)

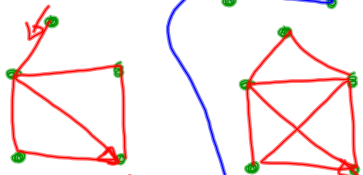


Try to trace out all of the edges of this graph without repeating any edges or lifting your pencil to start someplace new.



degrees of vertices



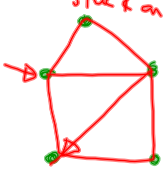


stuck on 3

start on 3

end on 3

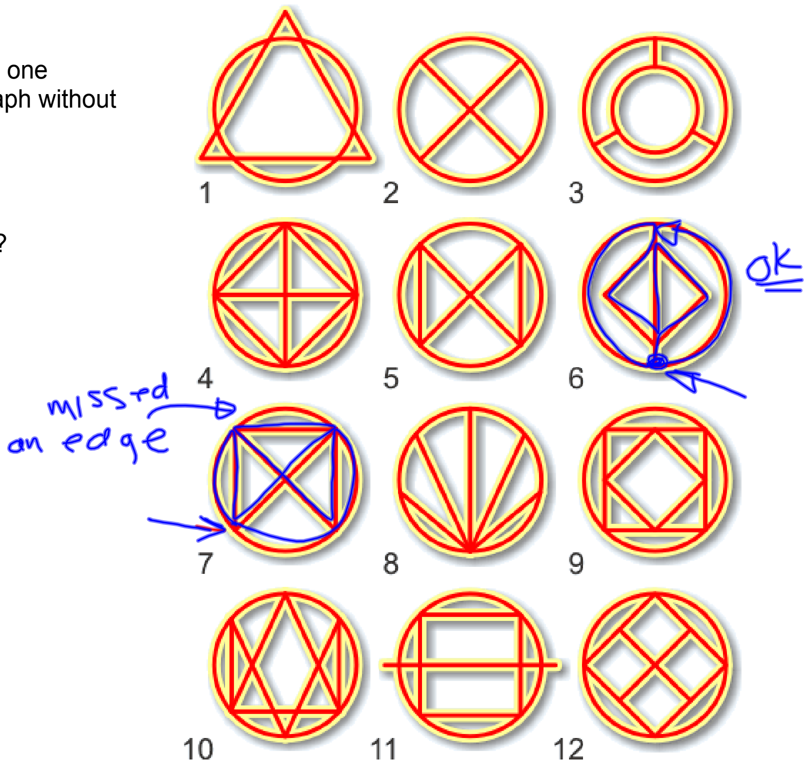
Start on 4

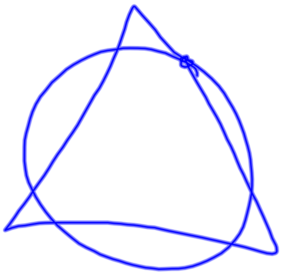


got stuck on a 3

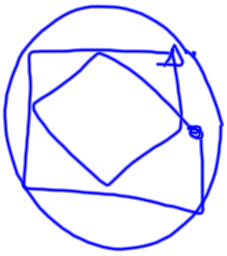
A Unicursal Drawing of a graph is one which draws each edge of the graph without going over the same edge twice.

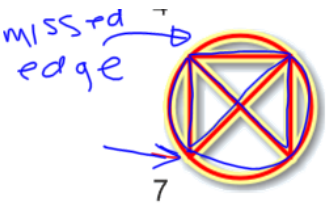
Which of these graphs have one?

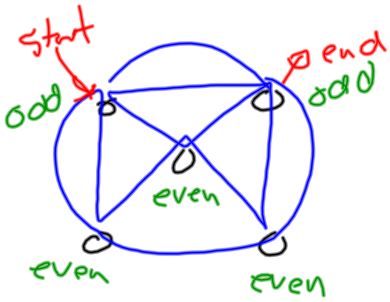


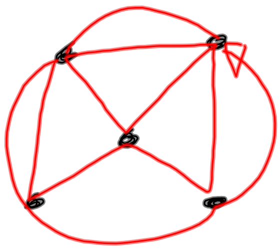


9

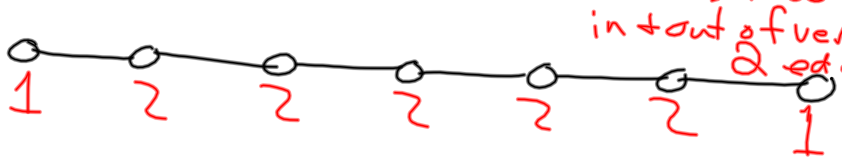








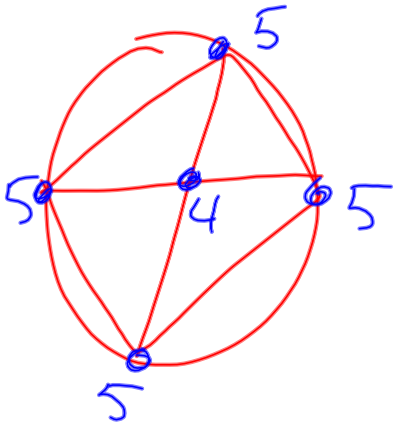
as I do the walk. in + out of vertex uses 2 edges.



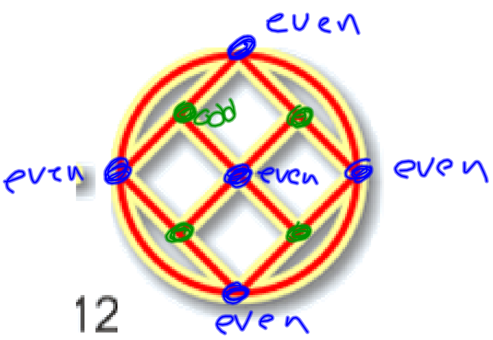
Have to start on odd, have to end on odd.

every vertex in middle must have even degree.

4



Doesnt Work.



12

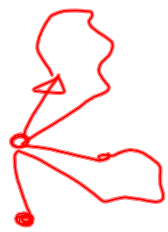
Impossible more than 2 "odd" vertices.

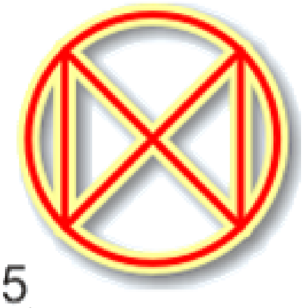
Can I draw this in one path?

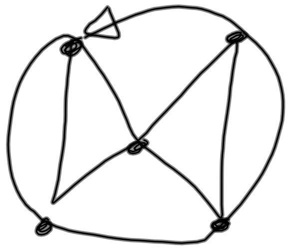
Starting vertex uses odd # of edges

Ending vertex uses odd # edges

every other vertex must have even # edges

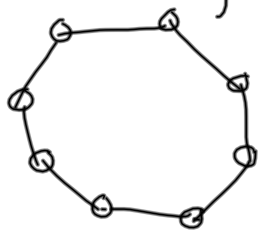






5

all vertices are even
all have 4 edges.

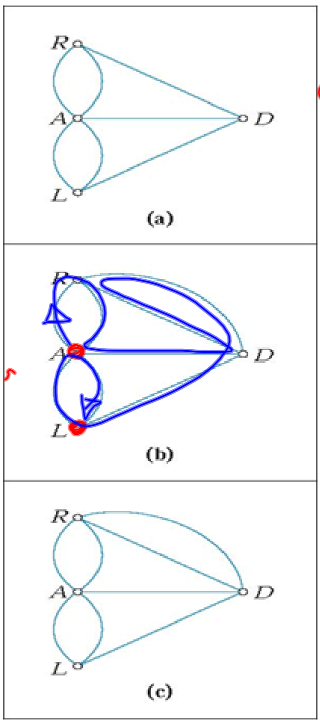


all even, start anywhere
get circuit
(not path)

An **Euler path** is a path that passes through *every* edge of a graph *once and only once*. The graph shown in (a) does not have an Euler path; the graph in (b) has several Euler paths. One of them is L,A,R,D,A,R,D,L,A.

4 odd vertices
No Path

2 odd vertices



http://www.flashandmath.com/mathlets/discrete/graphtheory/euler.html

Euler's Circuit Theorem

- If a graph is *connected*, and every vertex is *even*, then it has an Euler circuit (at least one, usually more).
- If a graph has *any* odd vertices, then it does not have an Euler circuit.

Start & end on same vertex.

Euler's Path Theorem

- If a graph is *connected*, and has exactly *two* odd vertices, then it has an Euler path (at least one, usually more). Any such path must start at one of the odd vertices and end at the other one.
- If a graph has *more than two* odd vertices, then it cannot have an Euler path.

Start and end on different vertices.

Attachments

- Web Pages as Graphs
- Euler Circuit
- TheHousesAndUtilitiesCrossingProblem.nbp