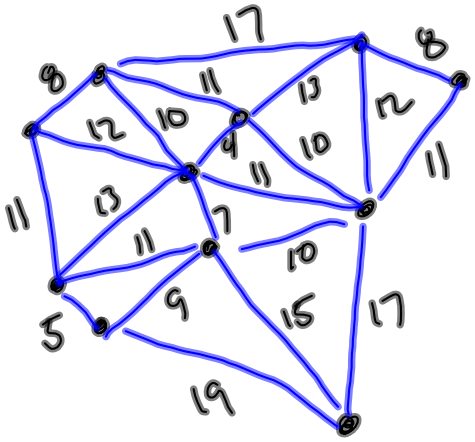
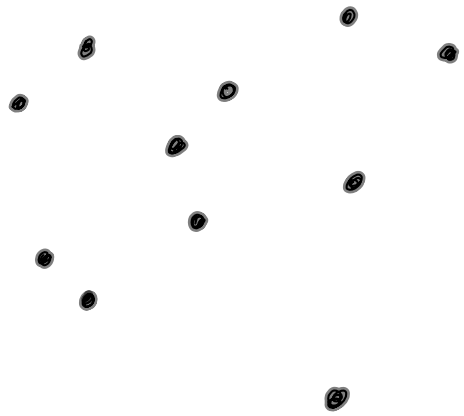
Network to Find Minimum Cost Spanning Tree

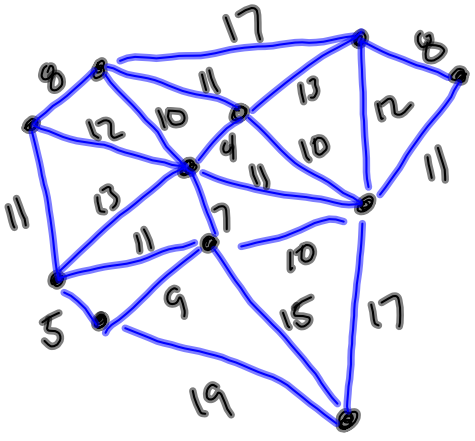
Use Kruskal's Algorithm

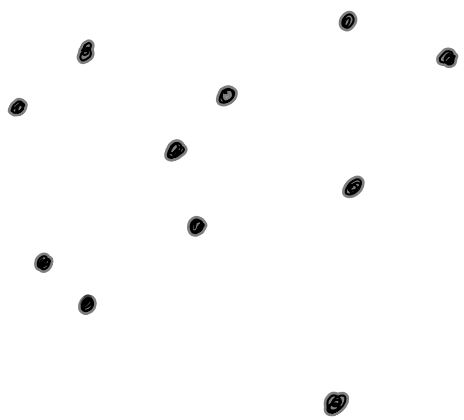




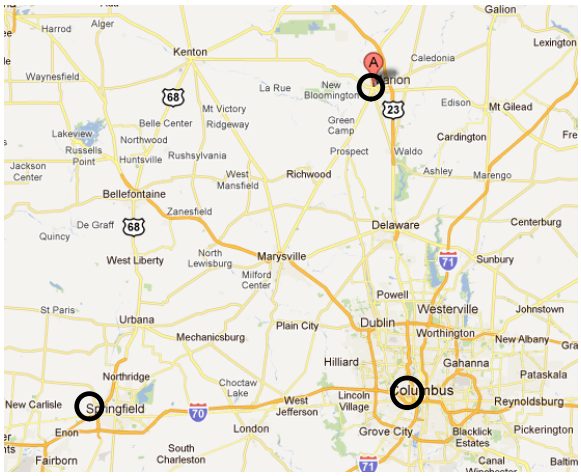
Network to Find Minimum Cost Spanning Tree

Use Prim's Algorithm

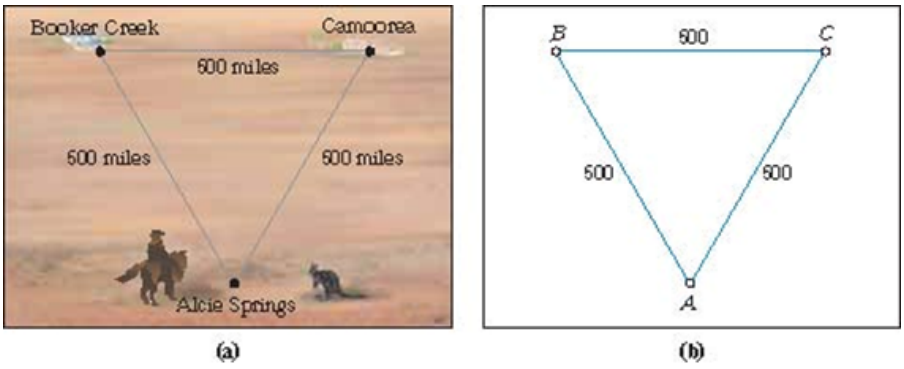




Where would you live to minimize the total distance of the three commutes if you had to travel from your home to Columbus, Springfield and Marion? If you had a cool jet-pack and didn't have to follow any roads....



http://www.math.osu.edu/~maharry/GeoGebra/StanSteinerPointsOnThreeAndFourPointNetworks.html



What is the shortest total distance of underground fiber-optic cable network needed to connect the three towns?

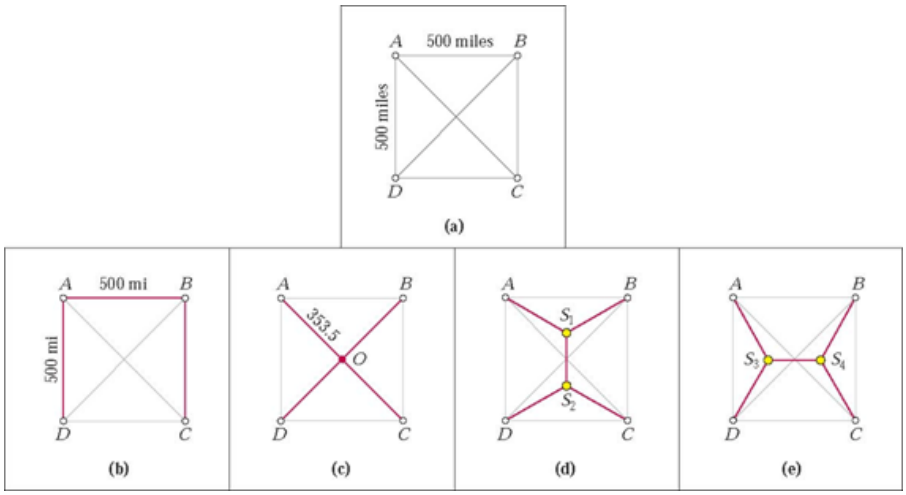
The search for the shortest network often starts with a look at the minimum spanning tree. The MST can always be found using Kruskal's algorithm and it gives us a ceiling on the length of the shortest network.

The main difference being that we can add new "junction vertices" if it reduces the overall length of the network.

There are three important terms that we will use in connection with junction points:

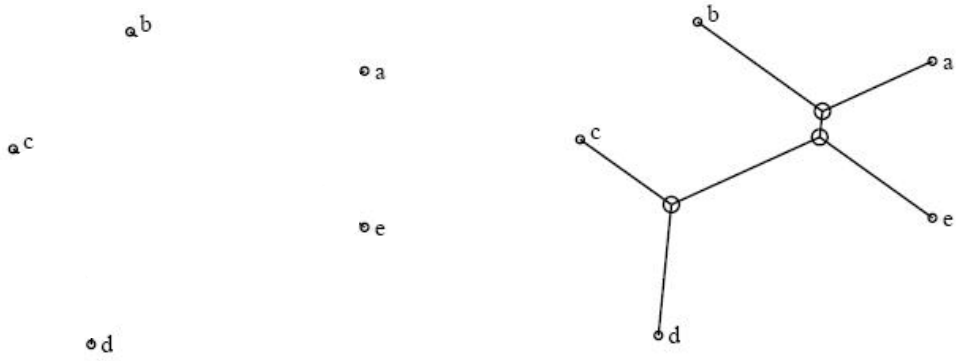
- In a network connecting a set of vertices, a junction point is said to be **native** junction point if it is located at one of the vertices.
- A nonnative junction point located somewhere other than at one of the original vertices is called an **interior** junction point of the network.
- An interior junction point consisting of three line segments coming together forming equal 120 angles
- An interior junction point consisting of three line segments coming together forming equal 120 angles (a "perfect" Y-junction if you will) is called a **Steiner point** of the network.

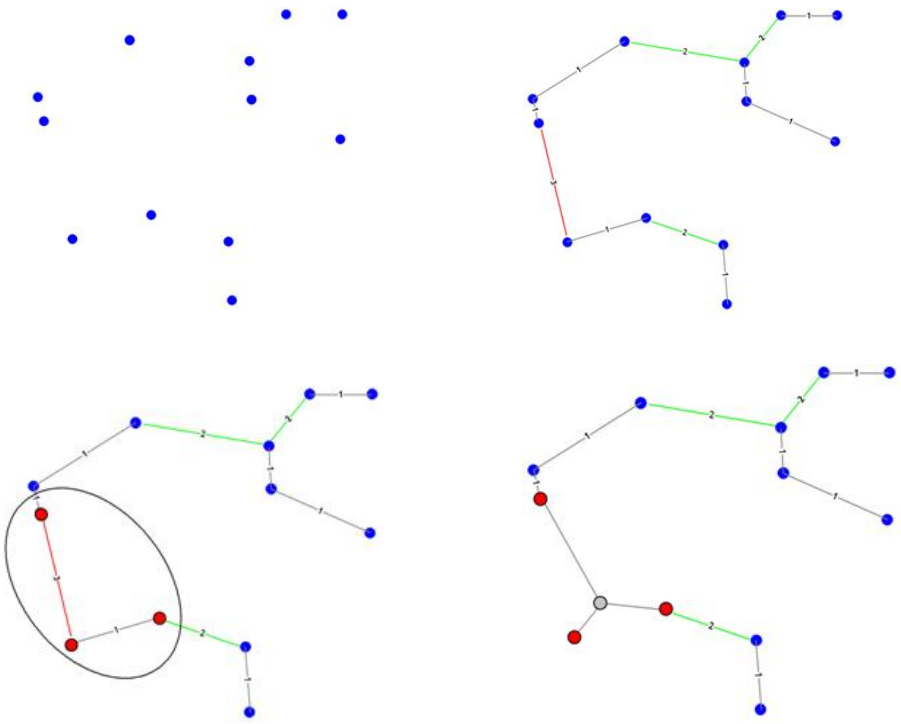
The only possible interior junction points in a shortest network are Steiner points. For convenience, we will call this **the interior junction rule for shortest networks**.



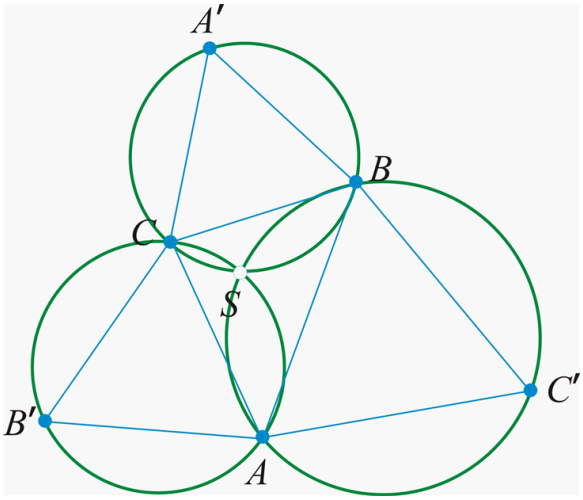
The Shortest Network Rule

- a minimum spanning tree (no interior junction points) or
- A Steiner tree. [**A Steiner tree** is a network with no circuits (a tree) such that all interior junction points are Steiner points.]





Finding Steiner Points: Torricelli's Construction



http://en.wikipedia.org/wiki/Evangelista_Torricelli