Math 1116
Probability Lecture
Monday-Wednesday 10:10-11:30

Course Web Page

http://www.math.ohio-state.edu/~maharry/

Chapter 15
Chances, Probabilities and Odds

Objectives

- To describe an appropriate sample space of a random experiment.
- To apply the multiplication rule, permutations, and combinations to counting problems.
- To understand the concept of a probability assignment.
- To identify independent events and their properties.
- To use the language of odds in describing probabilities of events.

Definitions

- **Random experiment**

  Description of an activity or process whose outcome cannot be predicted ahead of time.

Examples: Tossing Coins, Rolling Dice, Playing Cards, Elections, Bets etc.

- **Sample space**

  Associated with every random experiment is the set of all of its possible outcomes.

We will consistently use the letter S to denote a sample space and
N to denote its size (the number of outcomes in S).

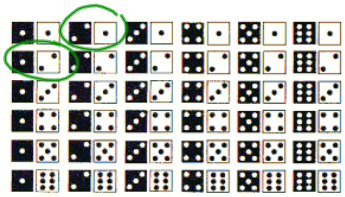
Sample Space.

Sample Space: Possible Values of Total when you roll 2 dice

2, 3, 4, 5, 6
7, 8, 9, 10, 11, 12 = S

N = 11 possible outcomes

Sample Space: Possible outcomes when you roll 2 dice



N = 36 possible outcomes

Question: How big is the sample space if you roll 3 dice?

N = 6 × 6 × 6 = 216

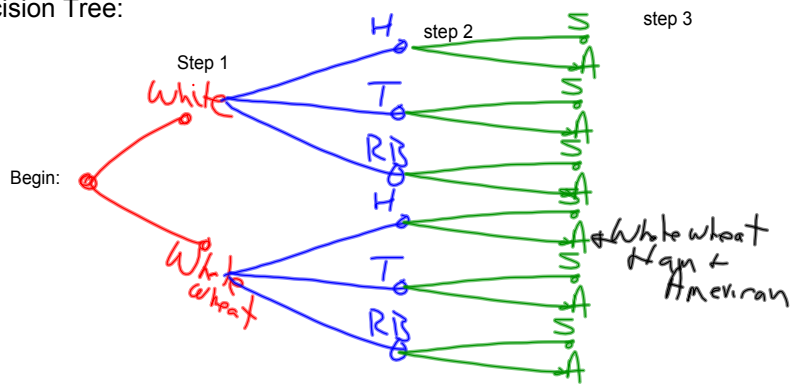
The Multiplication Rule

When something is done in stages, the number of ways it can be done is found by multiplying the number of ways each of the stages can be done.

How many different types of sandwiches can be made if there are
2 types of bread (white or wheat),
3 types of meat (ham, turkey, roast beef)
and 2 types of cheese (swiss, american)?

N = 2 · 3 · 2 = 12 ways

Decision Tree:



Examples: How many possible outcomes are there when you roll two dice?

How many possible outcomes are there when you roll <u>three</u> dice?

How many ways are there to choose an outfit if you have 3 pairs of shoes, 4 pairs of pants, and 7 shirts?

$N = 3 \cdot 4 \cdot 7 = 84$

How many ways are there to choose an outfit if you have 3 pairs of shoes, 4 pairs of pants, and 3 casual shirts, 4 dress shirts and 5 ties (only worn with dress shirts)?

$N = 3 \cdot 4 \cdot 3 \cdot 4 \cdot 5$

shoes pants

$N = 3 \cdot 4 \cdot (3 + 4 \cdot 5) = 276$

casual fancy

- **Permutation**

A group of objects where the ordering of the objects within the group makes a difference.
(Think of permuting the objects in all possible orders (different orders count as different outcomes))

How many ways can you rank your 5 favorite professors?

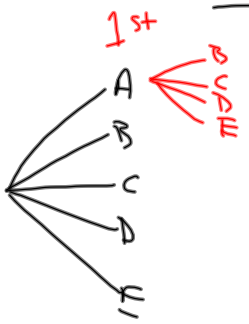
A.B.C.D.E

Put them in order How many ways?

A
B
C
D
E

N= 1st place · 2nd place · 3rd place · 4th last

N= 5 options · 4 options · 3 · 2 · 1



N= 120

N= 5!

"five factorial"

just 3 professors

A B C

How many rankings?

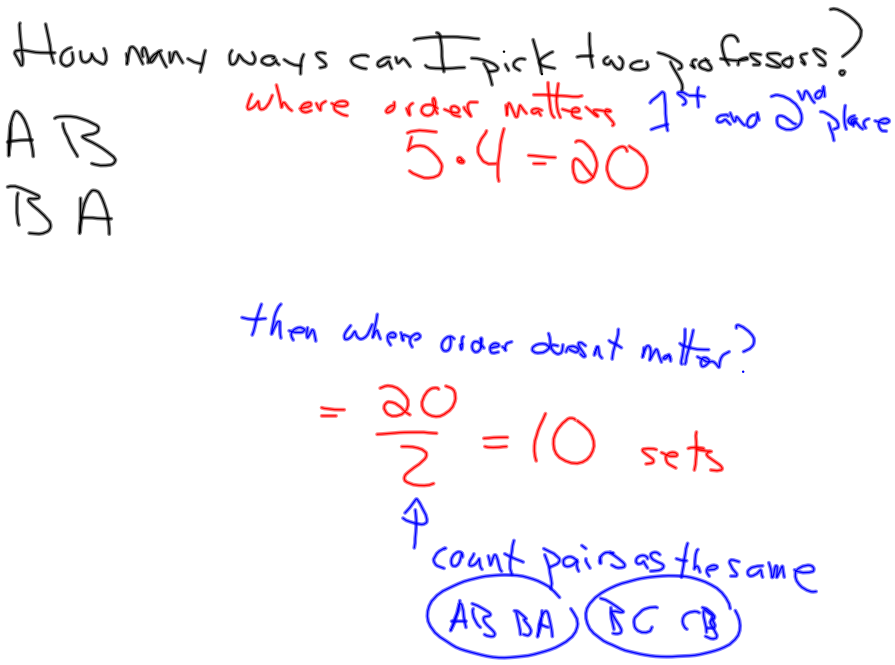
N= 3 · 2 · 1 = 6 = 3!

A B C
A C B
B A C
B C A
C A B
C B A

- **Combination**

A group of objects in which the ordering of the objects is irrelevant.

How many ways can you select two professors from the group of 5?



How many ways are there to put 'n' objects in order?

5 objects $N = 5\cdot4\cdot3\cdot2\cdot1 = 120$ possible orders

20 objects $N = 20\cdot19\cdot18\cdot17\cdots\cdots4\cdot3\cdot2\cdot1 = 20!$ "twenty factorial"

2.432902e+18

Factorials $n! = n\cdot(n-1)\cdot(n-2)\cdots\cdots3\cdot2\cdot1$ 2 with 18 zeros

$2! = 2\cdot1 = 2$

$3! = 6$

$4! = 24$

$5! = 5\cdot4\cdot3\cdot2\cdot1 = 120$

$= 5\cdot(4!)$

$6! = 720$

get big quickly....

Formulas for Permuations

How many permuations (order makes a difference) of r objects from a group of size n are there?

How many ways could somebody make a list of the best three football teams out of a group of 12 teams?

Rankings of all 12 teams
$= 12!$
huge

Rankings of only top 3 teams.
$N = 12 \cdot 11 \cdot 10$

$N = \frac{12 \cdot 11 \cdot 10 \cancel{\cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}}{\cancel{(9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)}}$

Divide by # of orders of 9 irrelavant teams

$N = \frac{12!}{9!}$

Deal 5 cards from Deck of 52.

5 cards in order

$N = 52 \cdot 51 \cdot 50 \cdot 49 \cdot 48$
1st 2nd 3rd 4th 5th

or

$N = \frac{52!}{47!}$

(all cards)
(all the other cards not dealt)

"permute"

Formula for # of ways to choose 'r' objects in order from a collection of size 'n'

r terms

$$ {}_nP_r = n\cdot(n-1)\cdot(n-2)\cdot\cdots\cdot(n-r+1) $$

$$ {}_nP_r = \frac{n\cdot(n-1)\cdot(n-2)\cdot\cdots\cdot(n-r+1)\cdot(n-r)!}{(n-r)!} $$

$$ {}_nP_r = \frac{n!}{(n-r)!} $$

(n-r) other objects that I don't care about.

$$ {}_{52}P_5 = 52\cdot 51\cdot 50\cdot 49\cdot 48 $$ (52-5+1)

How many groups of 3 teams can you pick from a collection of 12?

order doesn't matter

chose 3 teams from 12

$_{12}C_3$ = "Twelve choose 3"

"Combinations or Choose"

$$ N = \frac{12\cdot 11\cdot 10}{3\cdot 2\cdot 1} = 220 $$

groups of 6 equivalent orders

Possible orders

ABC or BDF or CFH

same sets 3·2·1 = 3!

ABC ACB BAC BCA CAB CBA

BDF BFD DBF DFB FBD FDB

3! equivalent orders

similar

Select a group of 4 teams from the 12 football teams

$$N = \frac{12 \cdot 11 \cdot 10 \cdot 9}{4 \cdot 3 \cdot 2 \cdot 1}$$

$$N = \frac{12!}{8!\,4!} = {}_{12}C_4$$

8! → 8 teams left out

4! → 4 teams included

# of 4 teams in order / # of equivalent orderings

ABCD

23 other orders count to the same

so divide by 24

Formula for # of ways to choose 'r' objects from a collection of size 'n'. combinations (Where order doesn't matter) Read it as "n choose r"

$$_nC_r = \frac{n!}{(n-r)!\,r!}$$

$$nCr = \frac{n!}{(n-r)!\,(r!)}$$

(n-r)! → orders of n-r objects left out

r! → orders of r objects taken

The local Ice Cream Shop advertises 31 flavors. How many ways can you pick three different flavors for a cone of ice cream? (strawberry on top is different than strawberry on the bottom)

Repeat Flavors
$N = 31 \cdot 31 \cdot 31$

$N = 31 \cdot 30 \cdot 29 = \frac{31!}{28!} = {}_{31}P_3$

Top middle bottom

The local Ice Cream Shop advertises 31 flavors. How many ways can you pick three different flavors for a bowl of ice cream?

order doesn't matter. cant repeat

$N = \frac{31 \cdot 30 \cdot 29}{3 \cdot 2 \cdot 1} = {}_{31}C_3$

(cant repeat) if I can repeat. its actually pretty complicated

How many ways are there to select a committee of 5 people (with President and a vice president) from a class of 23 people?

$N = (\text{pick 5 people}) \text{ then } (\text{from those 5 pick Pres and VP.})$

$N = ({}_{23}C_5) \cdot ({}_5P_2)$

5 people any order

2 people order matters

$N = \left(\frac{23!}{(18!)(5!)}\right)\left(\frac{5!}{3!}\right)$

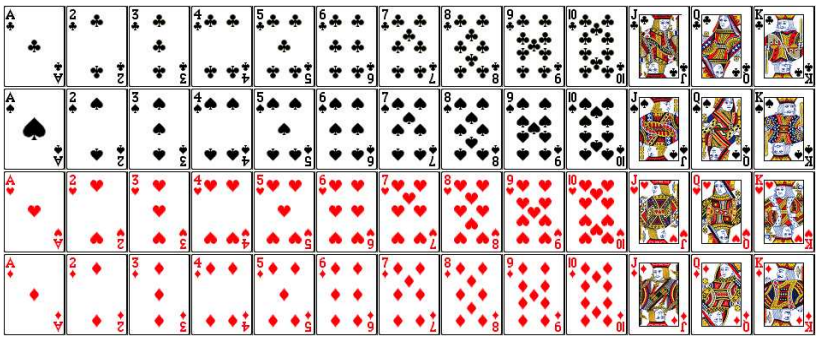
How many ways are there to select a committee of 5 people (with President and a vice president) from a class of 23 people?

Try the same problem another way

$N = ($ pick President $)($ Pick V.P. $)($ pick remaining 3 committee members $)$

Playing Poker

Suppose there are 52 cards in a deck and you are dealt a hand of 5 cards. How many possible ways can this happen?



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In Class Exercises and Examples:

3) The names of four people (A,B,C,D) are written on four slips of paper, put in a hat and mixed well. The slips are randomly taken out of the hat one at a time and the names recorded.
a) Write out the sample space for this random experiment. (Try to find a systematic way to do it)

b) Find N (the size of the sample space)

9) A California License plate starts with a digit other than 0, followed by three capital letters followed
by three more digits (0 to 9).
a) How many possible California License Plates are there?

b) How many start with a 5 and end with a 9?

c) How many have no repeated symbols?

15) A ski club at OSU has 35 members. Fifteen are female and 20 are male. A committee of four (President, V.P, Secretary and Treasurer) must be chosen.

a) How many different committees can be chosen?

b) How many different committees can be chosen if the President and Treasurer must be female?

15) A ski club at OSU has 35 members. Fifteen are female and 20 are male. A committee of four (President, V.P, Secretary and Treasurer) must be chosen.

c) How many different committees can be chosen if the President and Treasurer must be female and the V.P. and secretary must be male?

d) How many different committees can be chosen if there must be two females and two males?