Chapter 15 Notes on Probability

15.4 Probability Spaces

- **Probability assignment**

A function that assigns to each event E a number between 0 and 1, which represents the probability of the event E and which we denote by Pr (E).

- **Probability space**

Once a specific probability assignment is made on a sample space, the combination of the sample space and the probability assignment.

Elements of a Probability Space

- **Sample space**: $S = \{o_1, o_2, \ldots, o_N\}$
- **Probability assignment**: $Pr(o_1), Pr(o_2), \ldots Pr(o_N)$

[Each of these is a number between 0 and 1 satisfying $Pr(o_1) + Pr(o_2) + \ldots Pr(o_N) = 1$]

Events: These are all the subsets of S, including { } and S itself. The probability of an event is given by the sum of the probabilities of the individual outcomes that make up the event. [In particular, Pr({ }) = 0 and Pr(S) =1]

What could the weather be tomorrow?

S = { sunny, rainy, cloudy, snowy}

Pr({Sunny}) =

Pr({Cloudy}) =

Pr({Rainy}) =

Pr({Snowy}) =

Probabilities in Equiprobable Spaces

Pr(E) = k/N (where k denotes the size of the event E and N denotes the size of the sample space S).

A probability space where each simple event has an equal probability is called an **equiprobable** "equal opportunity" space.

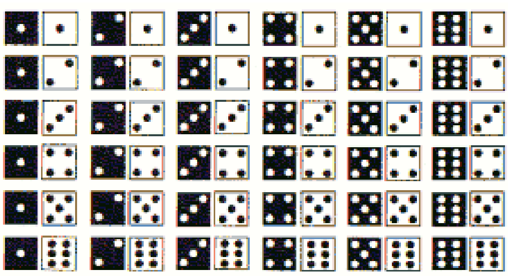
If all outcomes are equally likely,

Pr(Event) = (# Good Outcomes) / (Total # Outcomes)

Examples:

In the equiprobable space of rolling a pair of dice, find

Pr(Rolling doubles)



Pr(Rolling a value of 5)

Pr(Not Rolling Doubles)

Pr(Rolling at least 10)

Pr(Rolling at least one 6)

- **Independence Events**

If the occurrence of one event does not affect the probability of the occurrence of the other.

- **Multiplication Principle for Independent Events**

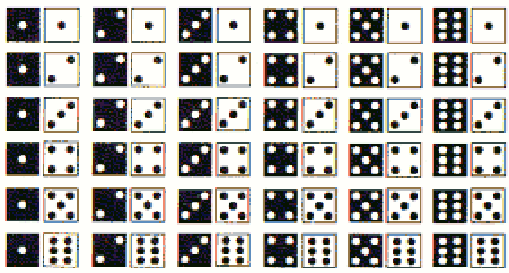
When events E and F are independent, the probability that both occur is the product of their respective probabilities; in other words,

$$Pr\ (E \text{ and } F) = Pr(E) \cdot Pr(F).$$

Find the probability of picking an ACE from a deck and then rolling a 3.

Pr =

Find the probability of rolling at least one 6.



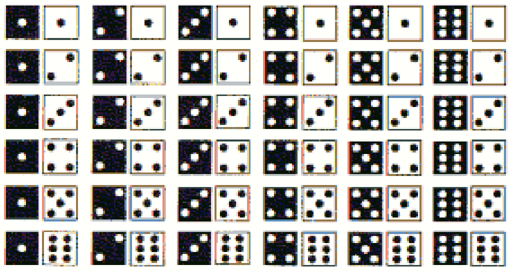
Pr(Exactly one six or two sixes)

Complementary Events:

If either E or F always happens (but never both), then the two events E and F are called complementary events.
The probabilities of complementary events add up to 1.
Thus, Pr(E) = 1 – Pr(F).

What is the complementary event to "rolling at least one six"?

Can you find the probability of that?



What are the probabilities of each of the following hands in 5 card poker?

(Think about how many ways you could set up the hand)

Pr(Four of a Kind)

Pr(Two Pair)

What is the probability that (at least) two people in this class have the same birthday?

Make a guess...
Do you think it is unlikely (close to zero) or almost certain (close to 1)?

Let's Check...

Jan	Feb
Mar	Apr
May	June
July	Aug
Sept	Oct
Nov	Dec

Make a Decision Tree for the sample space of flipping a coin four times.

How large is the sample space?

Pr(no heads) =
Pr(1 head) =

Pr(2 heads) =
Pr(3 heads) =
Pr(4 heads) =

What is the probability that in 10 coin flips you get 5 head and 5 tails? Make a guess.

Suppose you are shooting free throws and you know you have a 80% chance of making each one. Assume they are "independent".

Pr(Making three Free Throws in a row)

Pr(Making at least 2 out of 3 Free Throws)

Suppose there is a best-of-five series to determine the league champion.

Make a decision tree to find the possible outcomes. If each team has a 50/50 chance of winning each game, find Pr(5 game series)

Randomly choose 2 cards from a standard 52-card deck.
What is the probability of picking a pair of 5's?

Two ways to think about this.
First, as a two step process

Pr(pair of 5s) = Pr(first card is 5) * Pr(second card is 5 (given that first one was))

Second, as Good outcomes / total outcomes

Good Outcomes = How many ways can you have a pair of 5's?

Total Outcomes = How many ways can you pick two cards from the deck?

Randomly choose 3 cards from a standard 52-card deck.
What is the probability of NOT getting three-of-a-kind?

Randomly choose 3 cards from a standard 52-card deck.
What is the probability that all three cards are different suits?