Questions on Homework Problems or Handout from last Class

3. A computer password is made up of five characters. Each character can be a capital letter (A through Z) or a digit (0 through 9).

How many different such computer passwords are there?

- 5^{36}
- 36×5
- 36^5
- $26^5 + 10^5$
- None of the above

5 characters

~~26~~ ___ ___ ___ ___ ___

$$36 \cdot 36 \cdot 36 \cdot 36 \cdot 36 = 36^5$$

7. A computer password is made up of five characters. Each character can be a capital letter (A through Z) or a digit (0 through 9).

How many have 4 letters and only one digit?

- $4 \times 26 \times 10$
- $5 \times 26^4 \times 10$
- $26^4 \times 10$
- $26 \times 25 \times 24 \times 23 \times 10$
- None of the above

pick 4 letters

pick digit

$$N = 5 \cdot \underbrace{\underline{26}\ \underline{26}\ \underline{26}\ \underline{26}}_{\text{pick 4 letters}}\ \underline{10}$$

pick position for digit

8. *A computer password is made up of five characters. Each character can be a capital letter (A through Z) or a digit (0 through 9).*

How many have 3 letters and 2 digits?

- $26 \times 25 \times 24 \times 10 \times 9$
- $3 \times 26^3 \times 10^2$
- $26^3 \times 10^2$
- $10 \times 26^3 \times 10^2$
- *None of the above*

5 7 A K R

\# \# letter letter letter

N = (pick positions for 2 digits) · (pick 3 letters) (pick 2 numbers)

$$N = ({}_5C_2) \cdot (26 \cdot 26 \cdot 26) \cdot (10 \cdot 10)$$

$$\left(\frac{5!}{3! \cdot 2!}\right)(26^3)(10^2)$$

$$\boxed{N = 10 \cdot (26^3)(10^2)}$$

11. *110 golfers start a tournament. Assuming all of the golfers are equally skilled and that there are no ties, how many top 5 finishes are possible?*

- ${}_{110}C_5$
- 110×5
- $105!$
- ${}_{110}P_5$
- *None of the above*

Chapter 15 Notes on Probability

15.4 Probability Spaces

- **Probability assignment**

A function that assigns to each event E a number between 0 and 1, which represents the probability of the event E and which we denote by $\boxed{Pr(E)}$.

- **Probability space**

Once a specific probability assignment is made on a sample space, the combination of the sample space and the probability assignment.

all possible outcomes

→ Values assigned to each possible outcome.

Elements of a Probability Space

- **Sample space**: $S = \{o_1, o_2, \ldots, o_N\}$
- **Probability assignment**: $Pr(o_1), Pr(o_2), \ldots Pr(o_N)$

[Each of these is a number between 0 and 1 satisfying $Pr(o_1) + Pr(o_2) + \ldots Pr(o_N) = 1$]

Events: These are all the subsets of S, including { } and S itself. The probability of an event is given by the sum of the probabilities of the individual outcomes that make up the event. [In particular, Pr({ }) = 0 and Pr(S) =1]

$Pr(\{\ \}) = 0$

probability none of outcomes happen

$Pr(S) = 1$

probability that one of the possible outcomes happens.

What could the weather be tomorrow?

S = { sunny, rainy, cloudy, snowy}

Pr({Sunny}) = ~~.25~~ .20

Pr({Cloudy}) = .25 .60

Pr({Rainy}) = .25 .10

Pr({Snowy}) = .25 .10

Dont have to be equally likely.

Probabilites Do have to add up to 1.

Probabilities in Equiprobable Spaces

Pr(E) = k/N (where k denotes the size of the event E and N denotes the size of the sample space S).

A probability space where each simple event has an equal probability is called an **equiprobable** "equal opportunity" space.

Every simple event is equally likely.

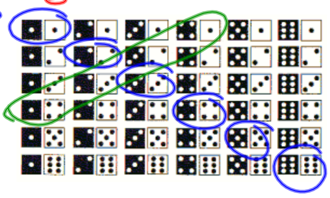
If all outcomes are equally likely,

Pr(Event) = (# Good Outcomes) / (Total # Outcomes)

Examples:

In the equiprobable space of rolling a pair of dice, find

Pr(Rolling doubles) $\frac{\#\text{ of doubles}}{\#\text{ of rolls}} = \frac{6}{36} = \frac{1}{6}$



Pr(Rolling a value of 5) $\frac{4}{36} = \frac{1}{9}$

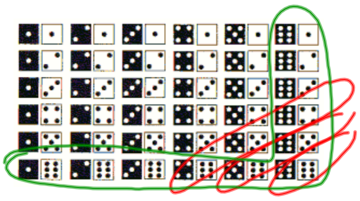
Pr(Not Rolling Doubles) $\frac{30}{36} = \frac{5}{6}$

Complement to rolling Doubles

Pr(Rolling at least 10) $6/36 = 1/6$

Pr(Rolling at least one 6)

$11/36$



Complement (Dont roll any 6's)

$Pr(\text{No } 6\text{'s}) = \frac{5 \cdot 5}{6 \cdot 6} = \frac{25}{36}$

- **Independence Events**

  If the occurrence of one event does not affect the probability of the occurrence of the other.

- **Multiplication Principle for Independent Events**

  When events E and F are independent, the probability that both occur is the product of their respective probabilities; in other words,

$$Pr(E \text{ and } F) = Pr(E) \cdot Pr(F).$$

$$\text{Prob}(\text{No 6's}) = \left(\frac{5}{6}\right)\cdot\left(\frac{5}{6}\right)$$

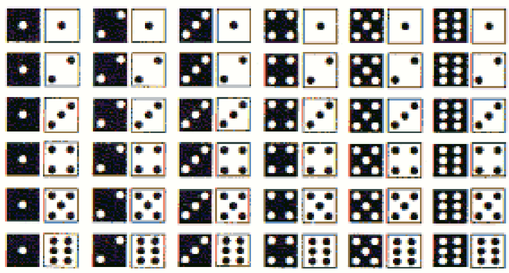
$$Pr\left(\begin{matrix}1^{st}\text{ die}\\ \text{Not } 6\end{matrix}\right)\cdot Pr\left(\begin{matrix}2^{nd}\text{ die}\\ \text{Not } 6\end{matrix}\right)$$

Find the probability of picking an ACE from a deck and then rolling a 3.

$$Pr = Pr\left(\begin{matrix}\text{pick}\\ \text{Ace}\end{matrix}\right)\cdot Pr\left(\begin{matrix}\text{roll a}\\ 3\end{matrix}\right)$$

$$= \underbrace{\left(\frac{4}{52}\right)\cdot\left(\frac{1}{6}\right)}_{\text{Final answer, don't need decimal (usually)}}$$

Find the probability of rolling at least one 6.



Pr(Exactly one six or two sixes)

Complementary Events:

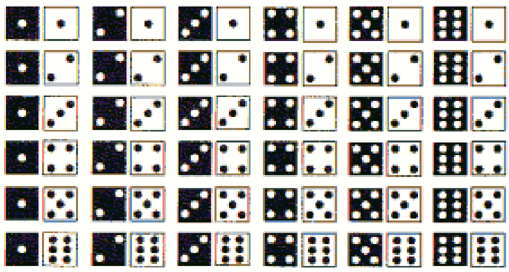
If either E or F always happens (but never both), then the two events E and F are called complementary events.

The probabilities of complementary events add up to 1.

Thus, Pr(E) = 1 – Pr(F).

What is the complementary event to "rolling at least one six"?

Can you find the probability of that?



What are the probabilities of each of the following hands in 5 card poker?

(Think about how many ways you could set up the hand)

$$\#\text{of possible Poker Hands} = \boxed{\frac{52\cdot 51\cdot 50\cdot 49\cdot 48}{5\cdot 4\cdot 3\cdot 2\cdot 1}}$$

pick 5 cards out of 52 = $\boxed{{}_{52}C_5}$

$$\text{Pr(Four of a Kind)} = \frac{\#\text{of 4 of a kind}}{\#\text{of possible Hands}} = \frac{(\text{pick rank first})(\text{pick last card})}{{}_{52}C_5}$$

$$= \frac{13\cdot(48)}{{}_{52}C_5}$$

)

Pr(Two Pair)

$$\left(\substack{\#\text{ of } 2\text{ pair}\\ \text{Hands}}\right) = \overset{1^{st}\text{ pair}}{(13 \cdot {}_4C_2)}\overset{2^{nd}\text{ pair}}{(12 \cdot {}_4C_2)}\overset{\text{last card}}{(44)}$$

(Not full House or 4 of kind)

$(13 \cdot 6)$

can't match 1st pair

Cards don't match either pair

Need to divide by two because order doesn't matter.

$$\left(\substack{\#\text{ of } 2\text{ pair}\\ \text{Hands}}\right) = \left(\frac{13 \cdot 12}{2}\right)({}_4C_2)({}_4C_2)(44)$$

Pick 2 ranks (order doesn't matter)

What is the probability that (at least) two people in this class have the same birthday?

Make a guess...
Do you think it is unlikely (close to zero) or almost certain (close to 1)?

Let's Check...

Jan	Feb
Mar	Apr
May	June
July	Aug
Sept	Oct
Nov	Dec

Make a Decision Tree for the sample space of flipping a coin four times.

How large is the sample space? $N = 2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$

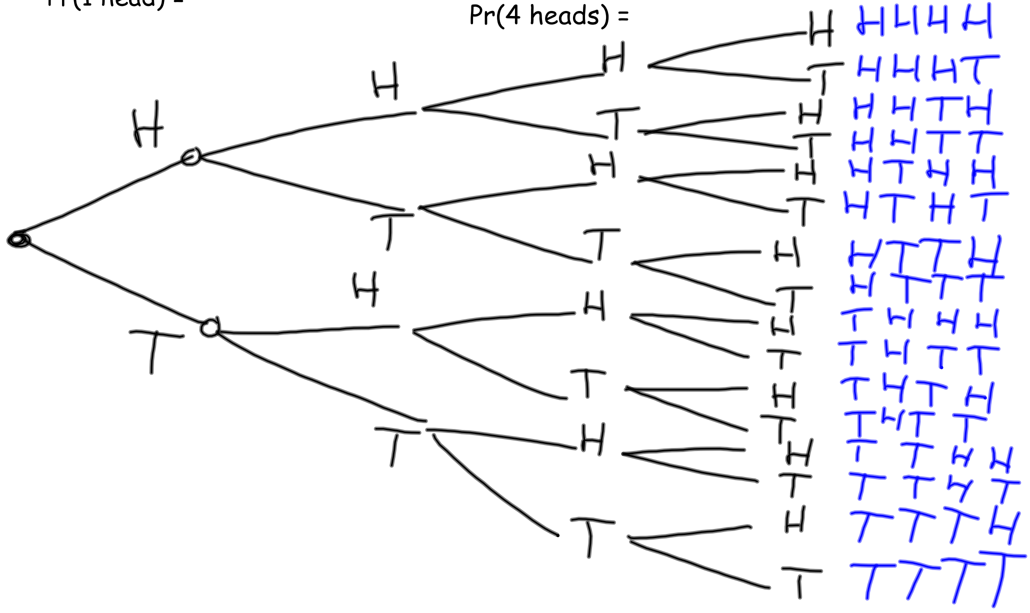
Pr(no heads) =

Pr(1 head) =

Pr(2 heads) =

Pr(3 heads) =

Pr(4 heads) =



What about 5 flips of a coin?

0 heads:

1 head:

2 heads

3 heads

4 heads

5 heads

Pascal's Triangle

http://www.mathsisfun.com/pascals-triangle.html

Try some small Combination

$$
\begin{array}{c}
1 \\
1 \quad 1 \\
1 \quad 2 \quad 1 \\
1 \quad 3 \quad 3 \quad 1 \\
1 \quad 4 \quad 6 \quad 4 \quad 1 \\
1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1 \\
1 \quad 6 \quad 15 \quad 20 \quad 15 \quad 6 \quad 1 \\
1 \quad 7 \quad 21 \quad 35 \quad 35 \quad 21 \quad 7 \quad 1 \\
1 \quad 8 \quad 28 \quad 56 \quad 70 \quad 56 \quad 28 \quad 8 \quad 1
\end{array}
$$

What is the probability that in 10 coin flips you get 5 head and 5 tails? Make a guess.

Suppose you are shooting free throws and you know you have a 80% chance of making each one. Assume they are "independent".

Pr(Making three Free Throws in a row)

Pr(Making at least 2 out of 3 Free Throws)

Suppose there is a best-of-five series to determine the league champion.

Make a decision tree to find the possible outcomes. If each team has a 50/50 chance of winning each game, find Pr(5 game series)

Randomly choose 2 cards from a standard 52-card deck.
What is the probability of picking a pair of 5's?

Two ways to think about this.
First, as a two step process

Pr(pair of 5s) = Pr(first card is 5) * Pr(second card is 5 (given that first one was))

Second, as Good outcomes / total outcomes

Good Outcomes = How many ways can you have a pair of 5's?

Total Outcomes = How many ways can you pick two cards from the deck?

Randomly choose 3 cards from a standard 52-card deck.
What is the probability of NOT getting three-of-a-kind?

Randomly choose 3 cards from a standard 52-card deck.
What is the probability that all three cards are different suits?

1. Suppose you are asked to create a password that consists of three letters (lower case) followed by 4 digits (0-9).

a. How many such passwords can be made? (i.e. kpp1939 is allowed). (Show the multiplications to find the answer, but don' actually multiply it out.)

$$N = \underline{26} \cdot \underline{26} \cdot \underline{26} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10}$$

a. What if you cannot repeat a letter or digit? (i.e. ksp1935 is allowed). How many are possible now?

$$N = 26\ 25\ 24\ 10 \cdot 9\ 8 \cdot 7$$
$$= \frac{26!}{23!} \cdot \frac{10!}{6!}$$
$$= {}_{26}P_3 \cdot {}_{10}P_4$$

a. What if the letters do not have to appear before the digits, but you still cannot repeat a symbol. (i.e. 4bc73k8 is allowed). How many passwords are possible now?

N = 3 letters 4 digits (any order) (No repeats)

$$\underset{\#}{\_\_}\ \underset{26}{\_\_}\ \underset{\#}{\_\_}\ \underset{\#}{\_\_}\ \underset{25}{\_\_}\ \underset{24}{\_\_}\ \underset{\#}{\_\_}$$

$$N = \underbrace{\left({}_7C_4\right)}_{\text{pick 4 spots out of 7}} \underbrace{(10 \cdot 9 \cdot 8 \cdot 7)}_{\text{\# of ways to pick digits}} (26 \cdot 25 \cdot 24)$$

same value

$${}_7C_3 = \frac{7!}{4! \cdot 3!}$$
$${}_7C_4 = \frac{7!}{3! \cdot 4!}$$

Chapter	Due Date	Walking	Jogging	Running
Ch. 15 Probability	Jan 23	2, 6, 10, 11, 13, 19, 20, 21, 22, 33, 43, 49, 50, 53, 56, 63	10, 11, 14, 16, 20, 34, 52, 56, 63, 66, 72, 73, 78	10, 16, 18, 24, 64, 69, 74, 76, 80, 83

11. Dolores packs two pairs of high-heel shoes, two pairs of tennis shoes, four formal dresses, three pairs of jeans, five T-shirts, and four silk blouses to go on a vacation.

(a) How many different "outfits" can Dolores make with these items? (Assume the following fashion standards: tennis shoes with a formal dress are not OK, tennis shoes with a silk blouse are not OK, high-heels with jeans and a T-shirt are not OK, high-heels with jeans and a silk blouse are fine.)