

Chapter 1

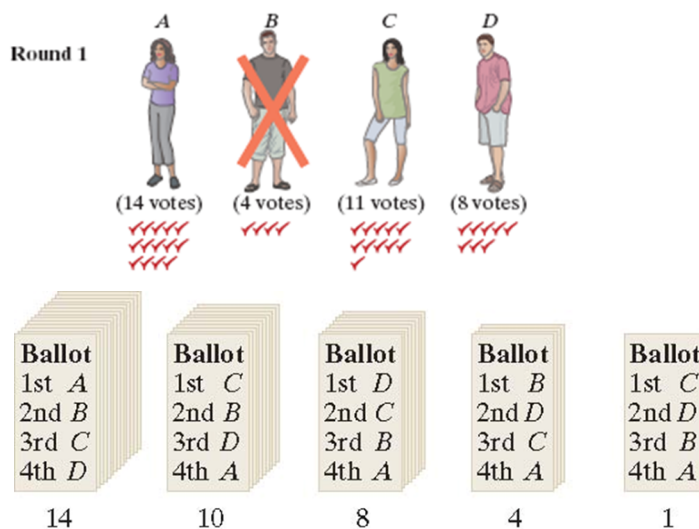
The Mathematics of Voting

Part II

Other Methods of Voting and Other "Fairness Criteria"

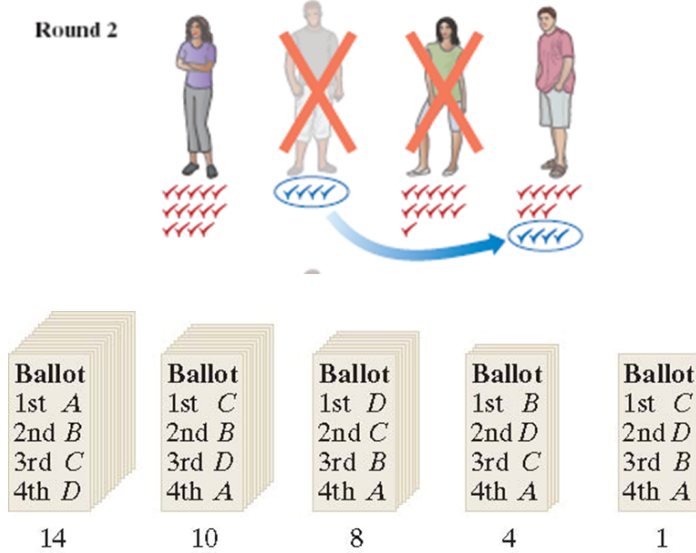
Plurality-with-Elimination Method

Round 1. Count the first-place votes for each candidate, just as you would in the plurality method. If a candidate has a majority of first-place votes, that candidate is the winner. Otherwise, eliminate the candidate (or candidates if there is a tie) with the *fewest* first-place votes.



Plurality-with-Elimination Method

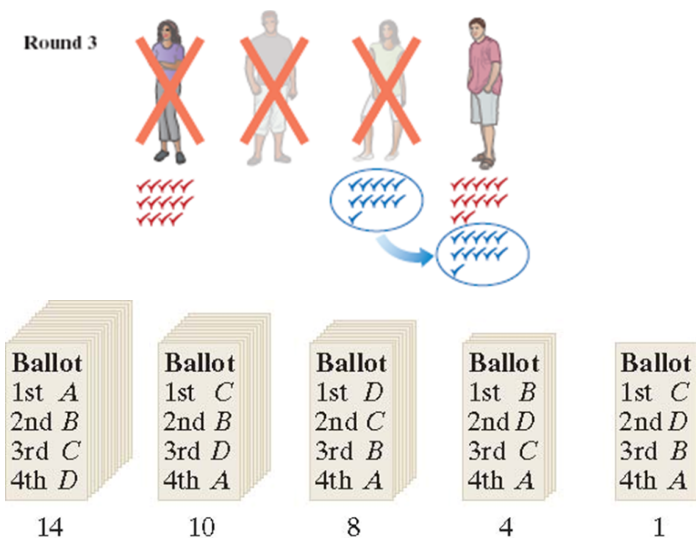
Round 2. Cross out the name(s) of the candidates eliminated from the preference and recount the first-place votes.



Plurality-with-Elimination Method

Round 3, 4, etc. Repeat the process, each time eliminating one or more candidates until there is a candidate with a majority of first-place votes.

That candidate is the winner of the election.



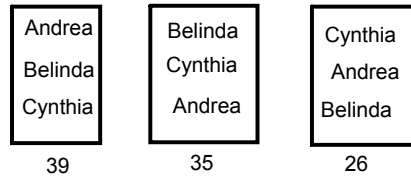
So what is wrong with the plurality-with-elimination method?

The Monotonicity Criterion

If candidate X is a winner of an election and, in a reelection, the only changes in the ballots are changes that favor X (and only X), then X should remain a winner of the election.

Monotonicity Criterion

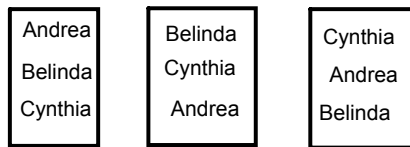
Suppose the votes are as follows:



Cynthia is eliminated, thus transferring votes to Andrea, who is elected with a majority.

Four years later...

She then serves a full term, and does such a good job that she persuades ten of Belinda's supporters to change their votes to her at the next election.



Who wins by Elimination in this election? Why does that seem wrong?

The Method of Pairwise Comparisons

In a pairwise comparison between between X and Y, every vote is assigned to either X or Y, *the vote going to whichever of the two candidates is listed higher on the ballot.* The winner is the one with the most votes; if the two candidates split the votes equally, it ends in a tie.

Number of voters	2	6	4	1	1	4	4
1st choice	A	B	B	C	C	D	E
2nd choice	D	A	A	B	D	A	C
3rd choice	C	C	D	A	A	E	D
4th choice	B	D	E	D	B	C	B
5th choice	E	E	C	E	E	B	A

Final Tally: A-3, B-2.5, C-2, D-1.5, E-1. (Choice A loses to B and beats C,D, and E) A wins.

So what is wrong with the method of pairwise comparisons?

The Independence-of-Irrelevant-Alternatives Criterion (IIA)

If candidate X is a winner of an election and in a recount one of the non-winning candidates is removed from the ballots, then X should still be a winner of the election.

Number of voters	2	6	4	1	1	4	4
1st choice	A	B	B	C	C	D	E
2nd choice	D	A	A	B	D	A	C
3rd choice	C	C	D	A	A	E	D
4th choice	B	D	E	D	B	C	B
5th choice	E	E	C	E	E	B	A

Eliminate C (an irrelevant alternative) from this election and B wins (rather than A).

Using our ballots from class for the presidential election.....

Romney = A
Paul = B
Gingrich = C
Obama = D

Who wins under each of the methods?

	2	3	3	1	1	1	1	1	1	1		(1)	(2)	(3)	(4)
1	A	B	A	B	D	A	D	B	C	B	A	6	42	7	4
2	B	D	C	A	A	B	B	A	D	D	B	6	45	8	6
3	C	A	B	C	C	C	C	D	B	C	C	1	31	0	2
4	D	C	D	D	B	D	A	C	A	A	D	2	32	0	0

Number of choices 4, number of distinct ballots 10

Plurality (1) Borda (2) Elimination (3) Pairwise (4) Reverse swap

Plurality =

Borda Count =

Elimination =

Pairwise Comparison

I changed the number of the 5th ballot to 6. And we get three different winners under the four methods

	2	3	3	1	6	1	1	1	1	1		(1)	(2)	(3)	(4)
1	A	B	A	B	D	A	D	B	C	B	A	6	57	0	4
2	B	D	C	A	A	B	B	A	D	D	B	6	50	12	3
3	C	A	B	C	C	C	C	D	B	C	C	1	41	0	1
4	D	C	D	D	B	D	A	C	A	A	D	7	52	8	4

Number of choices 4, number of distinct ballots 10

Plurality (1) Borda (2) Elimination (3) Pairwise (4) Reverse swap

How Many Pairwise Comparisons?

In an election between 5 candidates, there were 10 pairwise comparisons.

We could also count as an ${}_n C_r$ problem. How?

How many more comparisons would there be with 6 candidates?

Methods of Vote Counting


- Plurality
- Borda Count
- Plurality with Elimination
- Pairwise Comparisons
- Others

Fairness Criteria


- Majority Criterion
- Condorcet Criterion
- Monotonicity Criterion
- Independence of Irrelevant Alternatives Criterion
- Others

Arrow's Impossibility Theorem

It is mathematically impossible for a democratic voting method to satisfy all of the fairness criteria (in every possible case, when there are three or more candidates).

 [Wikipedia Voting Systems Page](#)

[Wikipedia Arrows Impossibility Theorem](#)

 [Wikipedia Page on Kenneth Arrow](#)



Nice Web Page to Compare Several Types of Voting Methods



Examples from Homework to Work on in Class. They all use the same preference ballots. Election for the chair of the Mathematics Department. Candidates Argand, Brandt, Chavez, Dietz, and Epstein.

3.

- How many people voted?
 - How many first place votes needed for majority?
 - Which candidate had most first place votes?
 - Which candidate had the most last place votes?
17. Find the winner under the Borda Count Method?

Number of Voters	5	3	5	3	2	3
1 st Choice	A	A	C	D	D	B
2 nd	B	D	E	C	C	E
3 rd	C	B	D	B	B	A
4 th	D	C	A	E	A	C
5 th	E	E	B	A	E	D

Examples from Homework to Work on in Class. They all use the same preference ballots. Election for the chair of the Mathematics Department. Candidates Argand, Brandt, Chavez, Dietz, and Epstein.

27.

- Find the winner under plurality with elimination method.

Number of Voters	5	3	5	3	2	3
1 st Choice	A	A	C	D	D	B
2 nd	B	D	E	C	C	E
3 rd	C	B	D	B	B	A
4 th	D	C	A	E	A	C
5 th	E	E	B	A	E	D

- Suppose that before the election, Chavez withdraws from the race. Find the winner under plurality with elimination.










Examples from Homework to Work on in Class. They all use the same preference ballots. Election for the chair of the Mathematics Department. Candidates Argand, Brandt, Chavez, Dietz, and Epstein.
27.

- Find the winner using the method of pairwise comparison.

Number of Voters	5	3	5	3	2	3
1 st Choice	A	A	C	D	D	B
2 nd	B	D	E	C	C	E
3 rd	C	B	D	B	B	A
4 th	D	C	A	E	A	C
5 th	E	E	B	A	E	D

Comparisons	against				
A	B	C	D	E	
B	C	D	E		
C	D	E			
D	E				
E					

Attachments

-  [Heisman Trophy Winner Selection](#)
-  [Alternate Voting Methods for Presidential Primaries](#)
-  [Results of Bush, Gore, Nader Presidential Vote in 2000](#)
-  [Wikipedia Article on Voting Methods and Criteria](#)
-  [Monotonicity Criterion](#)
-  [Wikipedia Voting Systems Page](#)
-  [wikipedia Arrows Impossibility Theorem](#)
-  [Wikipedia Page on Kenneth Arrow](#)
-  [Nice Web Page to Compare Several Types of Voting Methods](#)