The Method of Pairwise Comparisons

In a pairwise comparison between between *X* and *Y*, every vote is assigned to either *X* or *Y*, *the vote going to whichever of the two candidates is listed higher on the ballot*. The winner is the one with the most votes; if the two candidates split the votes equally, it ends in a tie.

Winner of Head to Head gets 1 point.

Tie both get 1/2 point.

Number of voters	2	6	4	1	1	4	4
1st choice	A	B	B	C	C	D	E
2nd choice	D	A	A	B	D	A	C
3rd choice	C	C	D	A	A	E	D
4th choice	B	D	E	D	B	C	B
5th choice	E	E	C	E	E	B	A

Most Points is winner

Final Tally: A-3, B-2.5, C-2, D-1.5, E-1. (Choice A loses to B and beats C,D, and E) A wins.

A vs B A vs C A vs D A vs E

A 2+1+4=7

B 6+4+1+4=15

Similar to Round Robin Tournament

Play every other candidate or team.

Count # of wins Head to Head.

Condorcet Winner

Wins every election Head to Head they should win.

Pairwise Comparison Method

If you win every election head to head you win by this method.

How Many Pairwise Comparisons?

In an election between 5 candidates, there were 10 pairwise comparisons.

AB AC AD AE

BC BD BE

CD CE

DE

1+2+3+4=10

If 6 candidates. How many Comparisons?

add FA FB FC FD FE

5 more games 1+2+3+4+5=15

We could also count as an ${}_nC_r$ problem. How?

$6C_2$ = # of Head to Head contests (pick 2 people from 6)

$= \frac{6!}{4!\cdot 2!} = \frac{6\cdot 5}{2\cdot 1} = 15$

How many more comparisons would there be with 6 candidates?

$1+2+3+\cdots+99$ = # of pairs out of 100 candidates

$100C_2 = \frac{100\cdot 99}{2\cdot 1} = 4950$

So what is wrong with the method of pairwise comparisons?

The Independence-of-Irrelevant-Alternatives Criterion (IIA)

If candidate *X* is a winner of an election and in a recount one of the non-winning candidates is removed from the ballots, then *X* should still be a winner of the election.

Number of voters	2	6	4	1	1	4	4
1st choice	*A*	*B*	*B*	~~*C*~~	~~*C*~~	*D*	*E*
2nd choice	*D*	*A*	*A*	*B*	*D*	*A*	~~*C*~~
3rd choice	~~*C*~~	~~*C*~~	*D*	*A*	*A*	*E*	*D*
4th choice	*B*	*D*	*E*	*D*	*B*	~~*C*~~	*B*
5th choice	*E*	*E*	~~*C*~~	*E*	*E*	*B*	*A*

Eliminate C (an irrelevant alternative) from this election and B wins (rather than A).

Do all pairwise comparisons of A B D & E.
B wins.

Methods of Vote Counting

- **Plurality** # of 1st place vote "most 1st place votes"
- **Borda Count** point system "most points"
- **Plurality with Elimination** Repeated Elects Eliminate fewest 1st place votes
- **Pairwise Comparisons** Head to Head "most wins wins"
- **Others** Round Robin

Fairness Criteria

- **Majority Criterion** more than half, I should win
- **Condorcet Criterion** beat everyone head to head, I should win.
- **Monotonicity Criterion** more people like me, I should still win.
- **Independence of Irrelevant Alternatives Criterion**
- **Others** Somebody drops out, winner should still win.

Arrow's Impossibility Theorem

It is mathematically impossible for a democratic voting method to satisfy all of the fairness criteria (in every possible case, when there are three or more candidates).

Wikipedia Voting Systems Page

Wikipedia Arrows Impossibility Theorem

Wikipedia Page on Kenneth Aarow

Attachments

- Heisman Trophy Winner Selection
- Alternate Voting Methods for Presidential Primaries
- Results of Bush, Gore, Nader Presidential Vote in 2000
- Wikipedia Article on Voting Methods and Criteria
- Monotonicity Criterion
- Wikipedia Voting Systems Page
- wikipedia Arrows Impossibility Theorem
- Wikipedia Page on Kenneth Aarow
- Nice Web Page to Compare Several Types of Voting Methods