Quesions on Homework on Voting Methods?

#70 2005 MVP Vote

Borda count with different point values.

How many points for 1st 2nd 3rd

	1st	2nd	3rd	
Pujols	18	14	0	378
Jones	13	17	2	351
Lee	1	1	30	263

Borda Count

Pujols $18(x)+14(y)+0(z)=378$

3 equations in 3 variables

Solve w/ Substitution.

Chapter 4 The Mathematics of Apportionment

How many representatives should each state have?

State or territory	Population estimate for July 1, 2012	Census population, April 1, 2010	Census population, April 1, 2000	Seats in U.S. House, 2013–2023
California	38,041,430	37,253,956	33,871,648	53
Texas	26,059,203	25,145,561	20,851,820	36
New York	19,570,261	19,378,102	18,976,457	27
Florida	19,317,568	18,801,310	15,982,378	27
Illinois	12,875,255	12,830,632	12,419,293	18
Pennsylvania	12,763,536	12,702,379	12,281,054	18
Ohio	11,544,225	11,536,504	11,353,140	16
Georgia	9,919,945	9,687,653	8,186,453	14
Michigan	9,883,360	9,883,640	9,938,444	14
North Carolina	9,752,073	9,535,483	8,049,313	13
New Jersey	8,864,590	8,791,894	8,414,350	12
Virginia	8,185,867	8,001,024	7,078,515	11
Washington	6,897,012	6,724,540	5,894,121	10
The Fifty States	313,281,717	308,143,815	280,849,847	435

For California:

$$\frac{37\,253\,958}{308\,143\,815} \times 435 = 52.59$$

round to 53

For Ohio

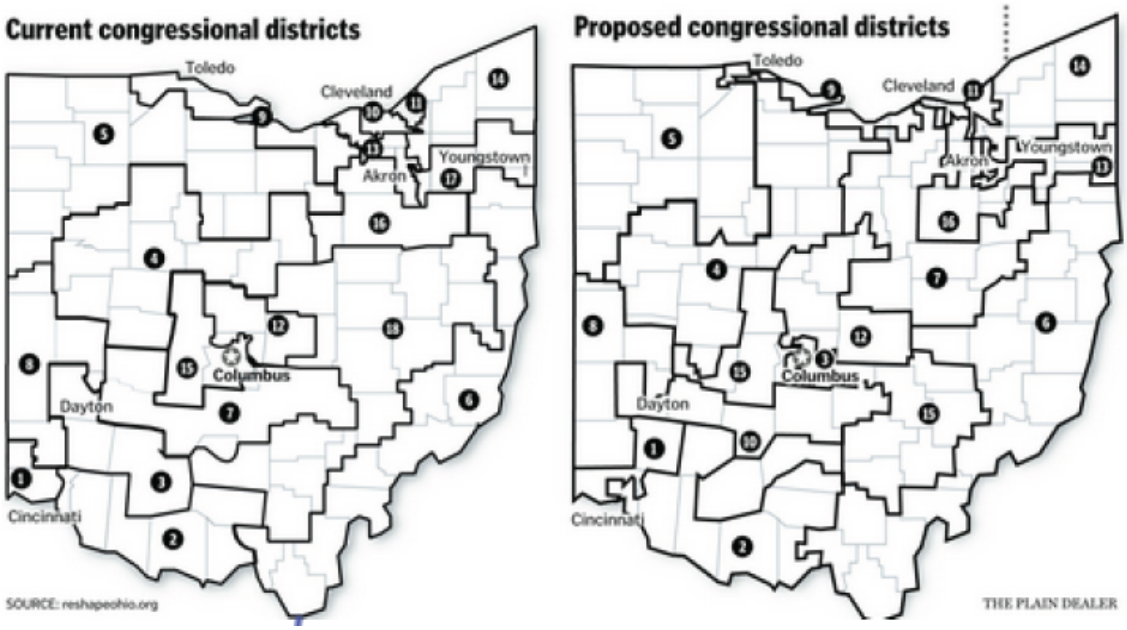
$$\frac{11\,536\,504}{308\,143\,815} \times 435 = 16.29$$

rounds to 16

in 2000 census

$$\frac{11\,353\,140}{280\,849\,847} \times 435 = 17.58$$

round to 18



18 Districts

16 Districts

We lost 2 districts in the last census. Why?

http://stateimpact.npr.org/ohio/tag/issue-2-2012/

Chapter 4
The Mathematics of Apportionment

Typical Problem

A school has one teacher available to teach all sections of Geometry, Precalculus and Calculus. She is able to teach 5 courses and no more.

How do you decide how many of each course should be offered?
How do you apportion the five sections to the three courses?

1st. It depends on how many students will be in each course.

There are 100 students in all. They are as follows:
- Geometry 52
- Precalculus 33
- Calculus 15

How would you assign her sections?

Kids per class

2 Geom	26
2 PreCalc	16 1/2
1 Calc	15

or

3 Geom	17 1/3
1 PreCalc	33
1 Calc	15

Which is better? More "fair"?

The first step is to find a good unit of measurement. The most natural unit of measurement is the ratio of students to sections. We call this ratio the **standard divisor** ***SD = P/M***

SD = 100/5 = *20 students per section* Avg size of class = 20

For example, take Geometry. To find a section's standard quota, we divide the course's population by the standard divisor:
Quota = population/SD = 52/20 = 2.6 52 students at 20 students per class

Geometry "should" have 2.6 sections... = 2.6 classes

Similarly, the quota for Precalcus is Pop/SD = $\frac{33 \text{ students}}{20 \text{ students/class}} = 1.65$ classes

Finally, the quota for Calculus is Pop/SD = Calc = $15/20 = .75$ classes

How do you "round" to make it fair?

Apportionment is the problem of rounding the quota to whole numbers in a way that is "fair" to everyone and satisfies the original problem. There are several ways to do this. None of which is perfect, but some are better than others.

round to nearest integer

First guess: Round each of the quotas to the nearest whole number. What happens in this case?

Geometry: Quota = 2.6 Final Apportionment: 3

Precalculus: Quota = 1.65 Final Apportionment: 2

Calculus: Quota= .75 Final Apportionment: 1

What's wrong with that? Too many classes.

General Problem: Assign a number of "seats" to each of the "states" in proportion to the "population" of each state.

- **The "states."** This is the term we will use to describe the *players* involved in the apportionment. Geometry, Calc & PreCalc
- **The "seats."** This term describes the set of *M identical, indivisible objects* that are being divided among the *N* states. Assign # of seats
- **The "populations."** This is a set of *N* positive numbers which are used as the basis for the apportionment of the seats to the states. based on # of students

- **Upper quotas.** The quota rounded up and denoted by *U*..
- **Lower quotas.** The quota rounded down and is denoted by *L*.

In the unlikely event that the quota is a whole number, the lower and upper quotas are the same.

Another Example from the Book:

Table 4-3 Republic of Parador (Population by State)

Assign a number of seats in Congress to each of the following 6 states in proportion to their relative populations. There are 250 seats in the congress.

Find the Standard Quotient (Population per Seat)

Total Population / # of seats = 12,500000 people / 250 seats = 50,000 people per seat in congress

Make a guess the apportionment. Does it work?

State	A	B	C	D	E	F	Total
Population	1,646,000	6,936,000	154,000	2,091,000	685,000	988,000	12,500,000

Quotas 32.9 | 137.8 | 154000/50000 = 3.08 | 41.82 | 13.7 | 19.76

Round off 33 | 138 | 3 | 42 | 14 | 20 = 251

too many seats

How should you round?

Hamilton's Method

- Step 1. Calculate each state's standard quota.
- Step 2. Give to each state its *lower quota*.
- Step 3. Give the surplus seats to the state with the largest fractional parts until there are no more surplus seats.

Hamilton's Method of Apportionment
http://www.cut-the-knot.org/Curriculum/SocialScience/AHamilton.shtml

U.S. Constitution on Apportionment
http://www.archives.gov/exhibits/charters/constitution_transcript.html

Article. I.

Section. 1.

All legislative Powers herein granted shall be vested in a Congress of the United States, which shall consist of a Senate and House of Representatives.

Section. 2.

The House of Representatives shall be composed of Members chosen every second Year by the People of the several States, and the Electors in each State shall have the Qualifications requisite for Electors of the most numerous Branch of the State Legislature.

No Person shall be a Representative who shall not have attained to the Age of twenty five Years, and been seven Years a Citizen of the United States, and who shall not, when elected, be an Inhabitant of that State in which he shall be chosen.

Representatives and direct Taxes shall be apportioned among the several States which may be included within this Union, according to their respective Numbers, which shall be determined by adding to the whole Number of free Persons, including those bound to Service for a Term of Years, and excluding Indians not taxed, three fifths of all other Persons. The actual Enumeration shall be made within three Years after the first Meeting of the Congress of the United States, and within every subsequent Term of ten Years, in such Manner as they shall by Law direct. The Number of Representatives shall not exceed one for every thirty Thousand, but each State shall have at Least one Representative; and until such enumeration shall be made, the State of New Hampshire shall be entitled to chuse three, Massachusetts eight, Rhode-Island and Providence Plantations one, Connecticut five, New-York six, New Jersey four, Pennsylvania eight, Delaware one, Maryland six, Virginia ten, North Carolina five, South Carolina five, and Georgia three.

When vacancies happen in the Representation from any State, the Executive Authority thereof shall issue Writs of Election to fill such Vacancies.

The House of Representatives shall chuse their Speaker and other Officers; and shall have the sole Power of Impeachment.

Hamilton's Method worked out for our 6-state Congress Example

Each seat = 50,000 people

State	Population	Step1 Quota	~~Step 2~~ Lower Quota	Fractional parts	Step 3 Surplus	Hamilton apportionment
A	1,646,000	32.92	32 +1	0.92	First	33
B	6,936,000	138.72	138 +1	0.72	Last	139
C	154,000	3.08	3	0.08		3
D	2,091,000	41.82	41 +1	0.82	Second	42
E	685,000	13.70	13	0.70		13
F	988,000	19.76	19 +1	0.76	Third	20
Total	12,500,000	250.00	246	4.00	4	250

4 seats remaining

Rules that apportionments should follow:

The Quota Rule

No state should be apportioned a number of seats smaller than its lower quota or larger than its upper quota.

When a state is apportioned *a number smaller than its lower quota*, we call it a **lower-quota violation**;

when a state is apportioned a *number larger than its upper quota*, we call it an **upper-quota violation**.)

The most serious (in fact, the fatal) flaw of Hamilton's method is commonly know as the **Alabama paradox**.

In essence, the paradox occurs when an *increase in the total number of seats being apportioned, in and of itself, forces a state to lose one of its seats*.

After the 1880 census, C. W. Seaton, chief clerk of the United States Census Bureau, computed apportionments for all House sizes between 275 and 350, and discovered that Alabama would get 8 seats with a House size of 299 but only 7 with a House size of 300.

		With 10 seats		With 11 seats	
State	Size	Fair share	Seats	Fair share	Seats
A	6	4.286	4	4.714	5
B	6	4.286	4	4.714	5
C	2	1.429	2	1.571	1

Wikipedia Alabama Paradox

SD standard division

#people/seat = $\frac{14}{10 \text{ seats}} = 1.4$

$6/1.4 = 4.286$ seats is my Quota.

+1 round up

2 extra seats to give out go to A & B.

The Hamilton's method can fall victim to two other paradoxes called

The population paradox- when state A *loses* a seat to state B even though the population of A *grew at a higher rate* than the population of B.

TABLE 4-10 Intergalactic Congress: Apportionment of 2525

Planet	Population	Step 1	Step 2	Step 3	Apportionment
Alanos	150	8.3	8	0	8
Betta	78	4.3	4	0	4
Conii	173	9.61	9	1	10
Dugos	204	11.3	11	0	11
Ellisium	295	16.38	16	1	17
Total	900	50.00	48	2	50

Happened to Ohio in last census.

Quota round down add back in

# of seats total

TABLE 4-11 Intergalactic Congress: Apportionment of 2535

Planet	Population	Step 1	Step 2	Step 3	Apportionment
Alanos	150	8.25	8	0	8
Betta	78	4.29	4	1	5
Conii	181	9.96	9	1	10
Dugos	204	11.22	11	0	11
Ellisium	296	16.28	16	0	16
Total	909	50.00	48	2	50

same population gained a seat

increased population Lost a seat

The new-states paradox- that *the addition of a new state with its fair share of seats can, in and of itself, affect the apportionments of other states.*

TABLE 4-12 Metro Garbage Truck Apportionments

District	Homes serviced	Quota (*SD* = 1000)	Hamilton apportionment
Northtown	10,450	10.45	10
Southtown	89,550	89.55	90
Total	100,000	100.00	100

100 trucks for North + South

TABLE 4-13 Revised Metro Garbage Truck Apportionments

District	Homes serviced	Quota (*SD* ≈ 1002.38)	Hamilton apportionment
Northtown	10,450	10.42	11
Southtown	89,550	89.34	89
Newtown	5,250	5.24	5
Total	105,250	105.00	105

down 1 truck.

Add a new "state" and add enough tracks for them.

Changes the apportionment of other states.

Jefferson's Method

Step 1. Find a "suitable" divisor D.
A suitable or **modified divisor** is a divisor that produces and apportionment of exactly M seats when the quotas (populations divided by D) are *rounded down*.

Step 2. Each state is apportioned its *lower quota*.

Bad News- Jefferson's method can produce **upper-quota violations**!
To make matters worse, the upper-quota violations tend to consistently favor the larger states.

The apportionment method suggested by Alexander Hamilton was approved by Congress in 1791, but was subsequently vetoed by president Washington - in the very first exercise of the veto power by President of the United States. Hamilton's method was adopted by the US Congress in 1852 and was in use through 1911 when it was replaced by Webster's method.

Hamilton's Method (Round Down) on 6-State Congress
Decrease Divisor until Correct number of seats

TABLE 4-14 Rounding Down the Quotas: $SD = 50{,}000$ and $D = 49{,}500$

State	Population	Standard quota ($SD = 50{,}000$)	Lower quota	Modified quota ($D = 49{,}500$)	Jefferson apportionment
A	1,646,000	32.92	32	33.25	33
B	6,936,000	**138.72**	138	140.12	**140**
C	154,000	3.08	3	3.11	3
D	2,091,000	41.82	41	42.24	42
E	685,000	13.70	13	13.84	13
F	988,000	19.76	19	19.96	19
Total	12,500,000	250.00	246		250

Adams's Method

Step 1. Find a "suitable" divisor D.
A suitable or **modified divisor** is a divisor that produces and apportionment of exactly M seats when the quotas (populations divided by D) are *rounded up*.

Step 2. Each state is apportioned its *upper quota*.

Bad News- Adam's method can produce **lower-quota violations!**
We can reasonably conclude that Adam's method is no better (or worse) than Jefferson's method– just different.

Adams's Method (Round Up) on 6-State Congress
Increase Divisor until Correct number of seats

TABLE 4-15 Rounding Up the Quotas: $D = 50{,}500$, and $D = 50{,}700$

State	Population	Quota ($D = 50{,}500$)	Upper quota ($D = 50{,}500$)	Quota ($D = 50{,}700$)	Adams's apportionment
A	1,646,000	32.59	33	32.47	33
B	6,936,000	137.35	138	136.80	137
C	154,000	3.05	4	3.04	4
D	2,091,000	41.41	42	41.24	42
E	685,000	13.56	14	13.51	14
F	988,000	19.56	20	19.49	20
Total	12,500,000		251		250

Webster's Method
Step 1. Find a "suitable" divisor *D*.
Here a suitable divisor means a divisor that produces an apportionment of exactly *M* seats when the quotas (populations divided by *D*) are *rounded the conventional way*.

Step 2. Find the apportionment of each state by rounding its quota the conventional way.

Webster's Method Finding Suitable Divisor

TABLE 4-16 Convertional Rounding: $D = 50{,}100$

State	Population	Standard quota ($D = 50{,}000$)	Nearest integer	Quota ($D = 50{,}100$)	Webster's apportionment
A	1,646,000	32.92	33	32.85	33
B	6,936,000	138.72	139	138.44	138
C	154,000	3.08	3	3.07	3
D	2,091,000	41.82	42	41.74	42
E	685,000	13.70	14	13.67	14
F	988,000	19.76	20	19.72	20
Total	12,500,000	250.00	251		250

Daniel Webster proposed his apportionment method in 1832.It was adopted by the Congress in 1842, and then replaced by Alexander Hamilton's in 1852. It was again adopted in 1901 and reconfirmed in 1911. Finally, it was replaced by Huntington-Hill's method in 1941.

TABLE 4-18 Summary of the Four Methods

	Hamilton	Jefferson	Adams	Webster
Quota rule	No violations	Upper-quota violations possible	Lower-quota violations possible	Upper- and lower-quota violations possible
Alabama paradox	Possible	Not possible	Not possible	Not possible
Population paradox	Possible	Not possible	Not possible	Not possible
New-states paradox	Possible	Not possible	Not possible	Not possible
Bias in favor of	Large states	Large states	Small states	Neutral

Appendix 2000–2010 Apportionments of the House of Representatives

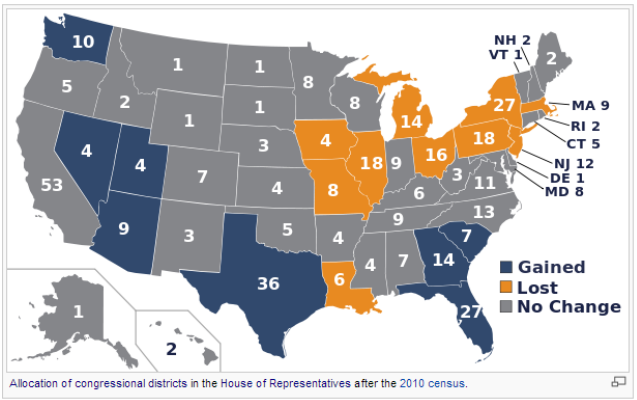
TABLE 1-9 Current Huntington-Hill Apportionments

State	Population	Seats	State	Population	Seats
Alabama	4,461,130	7	Montana	905,316	1
Alaska	628,933	1	Nebraska	1,715,369	3
Arizona	5,140,683	8	Nevada	2,002,032	3
Arkansas	2,679,733	4	New Hampshire	1,238,415	2
California	33,930,798	53	New Jersey	8,424,354	13
Colorado	4,311,882	7	New Mexico	1,823,821	3
Connecticut	3,409,535	5	New York	19,004,973	29
Delaware	785,068	1	North Carolina	8,067,673	13
Florida	16,028,890	25	North Dakota	643,756	1
Georgia	8,206,975	13	Ohio	11,374,540	18
Hawaii	1,216,642	2	Oklahoma	3,458,819	5
Idaho	1,297,274	2	Oregon	3,428,543	5
Illinois	12,439,042	19	Pennsylvania	12,300,670	19
Indiana	6,090,782	9	Rhode Island	1,049,662	2
Iowa	2,931,923	5	South Carolina	4,025,061	6
Kansas	2,693,824	4	South Dakota	756,874	1
Kentucky	4,049,431	6	Tennessee	5,700,037	9
Louisiana	4,480,271	7	Texas	20,903,994	32
Maine	1,277,731	2	Utah	2,236,714	3
Maryland	5,307,886	8	Vermont	609,890	1
Massachusetts	6,355,568	10	Virginia	7,100,702	11
Michigan	9,955,829	15	Washington	5,908,684	9
Minnesota	4,925,670	8	West Virginia	1,813,077	3
Mississippi	2,852,927	4	Wisconsin	5,371,210	8
Missouri	5,606,260	9	Wyoming	495,304	1
			Total	**281,424,177**	**435**

http://en.wikipedia.org/wiki/United_States_congressional_apportionment

http://en.wikipedia.org/wiki/List_of_U.S._states_and_territories_by_population

Gain more than one	Gain one		Lose one		Lose more than one
Texas +4 Arizona +2	Florida +1 Georgia +1 Nevada +1	Oregon +1 South Carolina +1 Utah +1	Illinois -1 Iowa -1 Louisiana -1 Massachusetts -1 Michigan -1	Minnesota -1 Missouri -1 New Jersey -1 New York -1 Pennsylvania -1	Ohio -2



Allocation of congressional districts in the House of Representatives after the 2010 census.

Attachments

- Hamilton's Method of Apportionment
- Section 2 of Constitution: Apportionment of Representatives
- Wikipedia Alabama Paradox
- Webster's Method Finding Suitable Divisor
- Projected Changes in Representatives 2010 Census
- State Populations as of 2008