The Traveling Salesman Problem

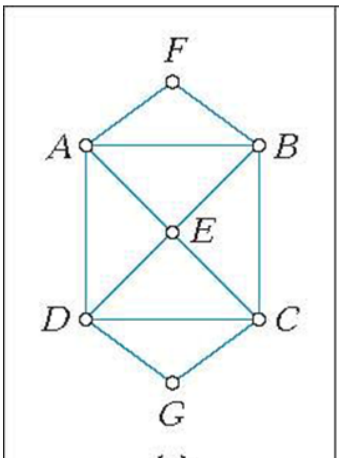
- **Hamilton path**
  A path that visits each vertex of the graph once and only once.
- **Hamilton circuit**
  A circuit that visits each vertex of the graph once and only once (at the end, of course, the circuit must return to the starting vertex).

http://en.wikipedia.org/wiki/William_Rowan_Hamilton

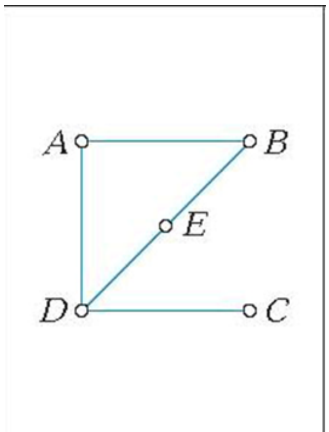
http://wps.prenhall.com/esm_tannenbaum_excursions_5/14/3687/943975.cw/index.html

Do the following graphs have
Euler Paths? Euler Circuits? Hamilton Paths? Hamilton Circuits?

If not, why not? If so, list them.



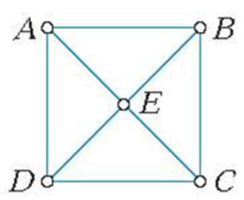
(a)



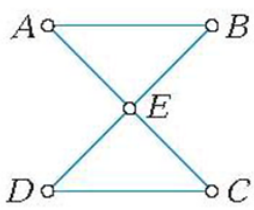
(b)

Do the following graphs have
Euler Paths? Euler Circuits? Hamilton Paths? Hamilton Circuits?

If not, why not? If so, list them.



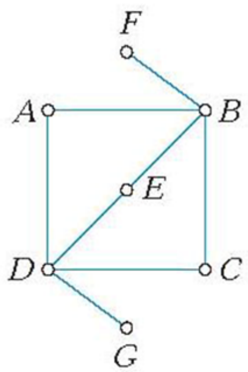
(c)



(d)

Do the following graphs have
Euler Paths? Euler Circuits? Hamilton Paths? Hamilton Circuits?

If not, why not? If so, list them.



(e)

Reasons why a graph might not have a Hamilton Circuit:

1. If the graph is disconnected

2. If the graph has a vertex of degree one.

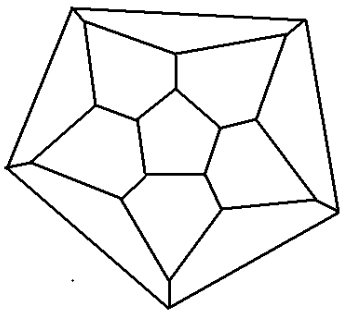
3. If the graph has an edge that is a bridge.

But there is no "nice" reason that explains when a graph has no Hamilton Circuit.

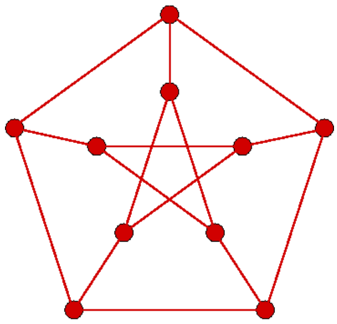
The only reason a connected graph has no Euler circuit is that it has odd vertices.

Examples of Graphs: Do they have a Hamilton Circuit?

Icosahedron

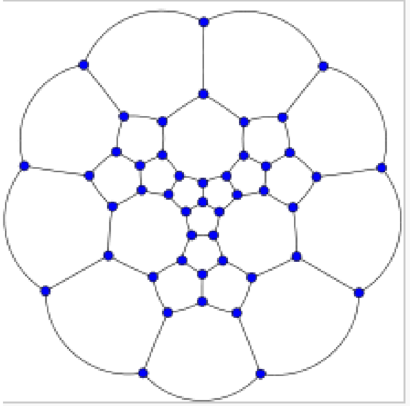


The Petersen Graph

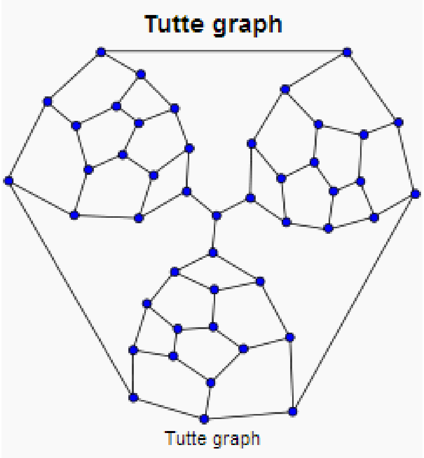


http://www.flashandmath.com/mathlets/discrete/graphtheory/graph2.html

Two nice planar Non-Hamiltonian Graphs



A graph that can be proven non-Hamiltonian using Grinberg's theorem



Tutte graph

http://en.wikipedia.org/wiki/W._T._Tutte

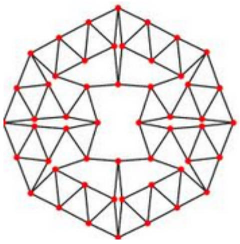
NP-Complete Problems

There are many famous algorithm questions (some about graphs, some about various other math problems) that are so-called NP-Complete.

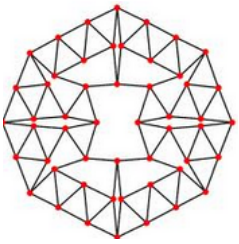
Nobody knows a "simple, quick" way to answer them without more or less trying every possible case. Like finding a hamilton circuit. However, if someone shows you a solution, it is easy to see that it works. Again like a Hamilton circuit.

Suppose somebody gives you a big graph (say 60 vertices). What do you have to check to see it it has an

Euler Circuit?



Hamilton Circuit?



http://en.wikipedia.org/wiki/NP-complete

There are, however, nice theorems that identify special situations where a graph must have a Hamilton circuit.

For instance, the previous graph must be Hamiltonian by:
Tutte's Theorem : Any 4-regular planar graph has a Hamiltonian circuit.

Another well-known known theorems is:
Dirac’s theorem:
If a connected graph has N vertices (N > 2) and all of them have degree bigger or equal to N / 2, then the graph has a Hamilton circuit.

But nothing is known to work for all graphs to decide if it has a Hamilton circuit or not, other than checking all possible circuits.

If a graph has a Hamilton circuit, then how many different Hamilton circuits does a it have?

A graph with *N* vertices in which *every* pair of distinct vertices is joined by an edge is called a **complete graph** on *N* vertices and denoted by the symbol K_N.

How many edges are there in a complete graph (no loops or multiple edges) with "n" vertices?

n=3

n=4

n=5

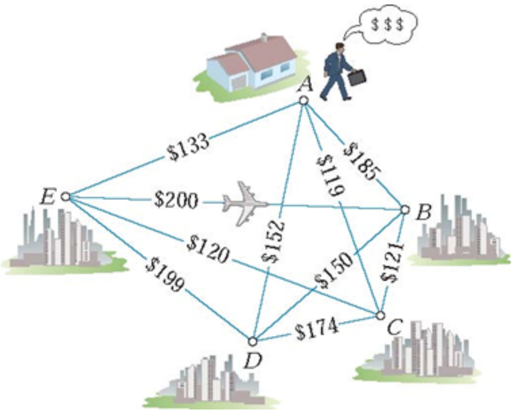
n=6

n

How many Hamilton Paths are there in a complete graph of order 5?

How many Hamilton Circuits are there in a complete graph of order 5?

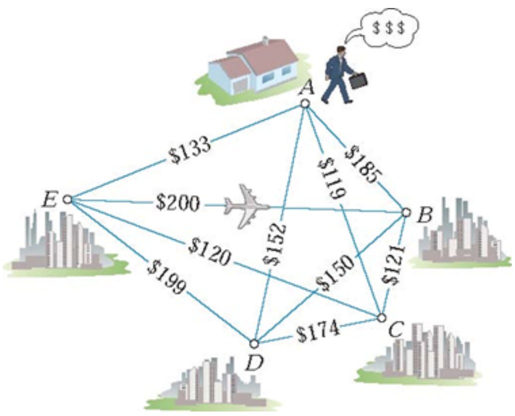
A weighted graph:



Travelling Salesman Problem (TSP)

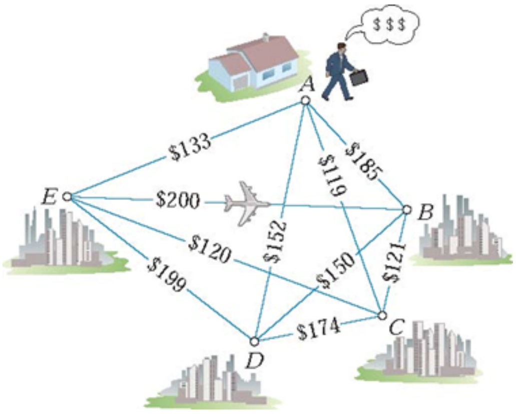
Find an optimal Hamilton circuit (a Hamilton circuit with least total weight) for the given weighted graph.

Algorithm:



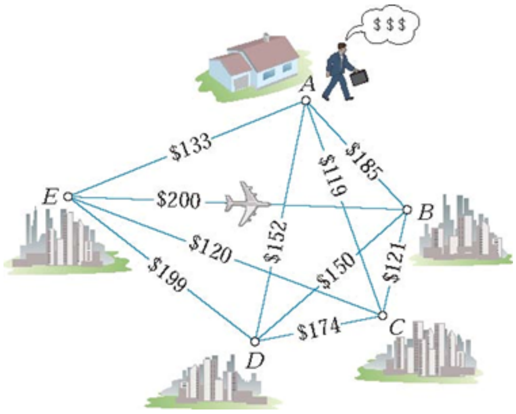
Strategy 1 (Exhaustive Search)

the **brute-force algorithm**



Strategy 2 (Go Cheap)

the **nearest-neighbor algorithm**



Algorithm 1: The Brute-Force Algorithm

- Step 1.
- Step 2.
- Step 3.

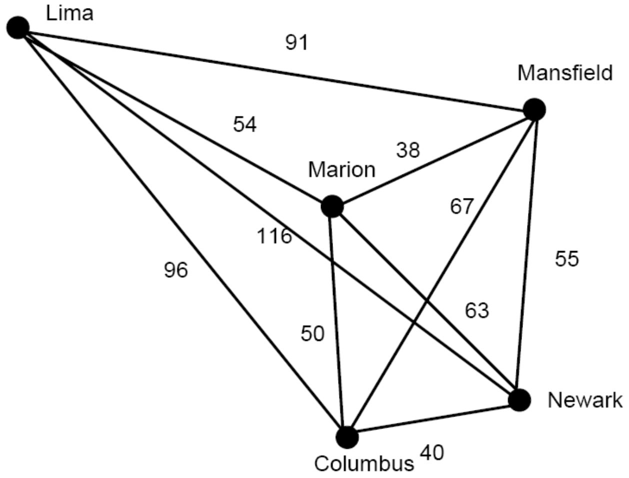
Algorithm 2: The Nearest-Neighbor Algorithm

Start.

First step.

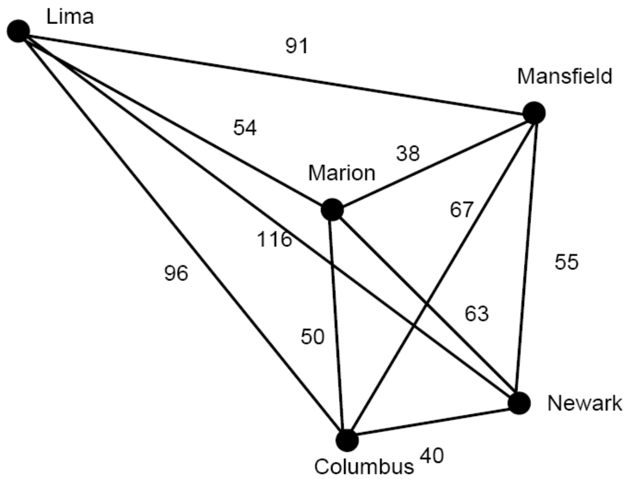
Middle steps.
Repeat this until all the vertices have been visited. Then take last edge back to starting vertex.

Suppose Ohio State's President Gee wants to visit all of the campuses: Columbus, Lima, Marion, Mansfield, a
Newark. He'd like to make one tour, and keeping his driving mileage to a minimum.



Find a Hamilton Circuit with Columbus as the start and finish using Nearest Neighbor.

Do the same thing except suppose you start and end in Newark. What happens?



The brute-force algorithm is an **Inefficient algorithm:**

How many steps for a 5-city problem?

How many for a 10-city problem?

How many for a 50-city problem?

The nearest-neighbor algorithm is an **efficient algorithm**.

How many edges do you have to check at each step in a 5-city problem (at most)?
How many steps are there?
How many total edge-checks are there?

What about a 50-city problem?

But....

A really good algorithm for solving TSP's in general would have to be both efficient (like the nearest-neighbor) and *optimal* (like the brute-force). Unfortunately, nobody knows of such an algorithm.

We will use the term **approximate algorithm** to describe any algorithm that produces solutions that are, most of the time, reasonably close to the optimal solution.

Algorithm 3: The Repetitive Nearest-Neighbor Algorithm

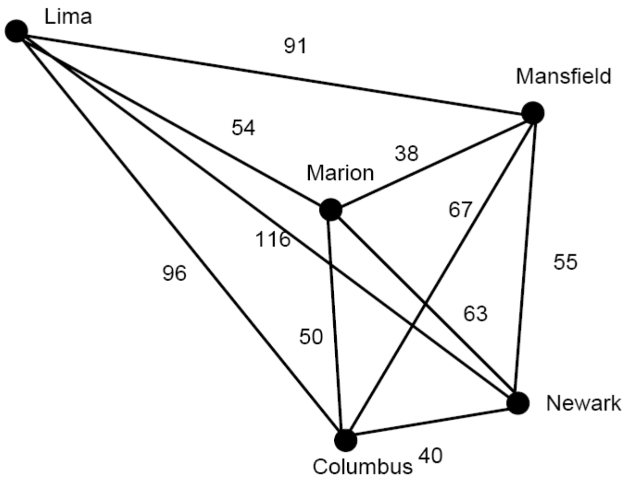
Start in Columbus

Start in Newark

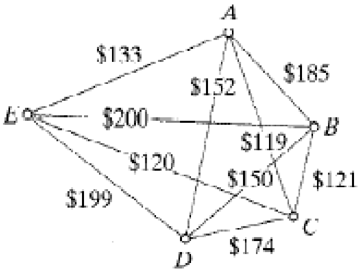
Start in Marion

Start in Mansfield

Start in Lima



31. For the weighted graph shown in the figure, (i) find the indicated circuit, and (ii) give its cost. (This is the graph discussed in Example 6.7.)

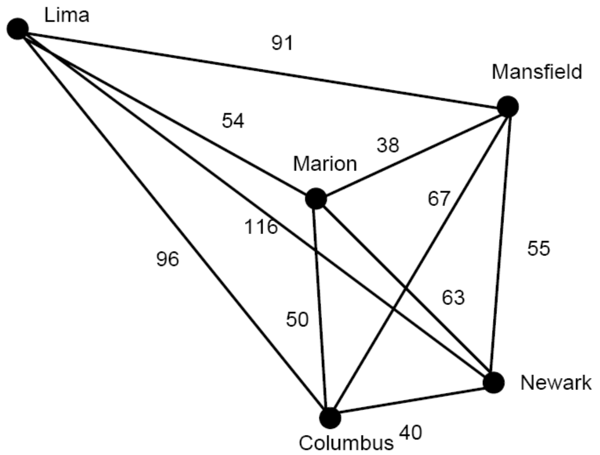


(a) The nearest-neighbor circuit for starting vertex *B*
(b) The nearest-neighbor circuit for starting vertex *C*
(c) The nearest-neighbor circuit for starting vertex *D*
(d) The nearest-neighbor circuit for starting vertex *E*

Algorithm 4: The Cheapest-Link Algorithm

Suppose Ohio State's President Holbrook wants to visit all of the campuses: Columbus, Lima, Marion, Mansfield, and Newark. She'd like to make one tour, and keeping her driving mileage to a minimum.

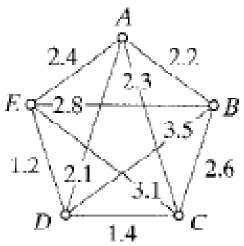
Find a Hamilton Circuit using Cheapest Link.



http://www-e.uni-magdeburg.de/mertens/TSP/TSP.html

Click on "Nearest Neighbor Heuristic"
Click "Run" or "Step" to do cheapest link
Then Reset and hit solve to show optimal circuit

43. For the weighted graph in the figure, find the Hamilton circuit obtained by the cheapest-link algorithm, and give the total mileage for this circuit. Write the circuit assuming that the starting and ending point is B. (This is the graph in Exercise 37.)



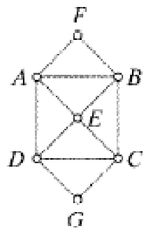
5. For the following graph,

(a) find a Hamilton path that starts at A and ends at E.

(b) find a Hamilton circuit that starts at A and ends with the edge EA.

(c) find a Hamilton path that starts at A and ends at C.

(d) find a Hamilton path that starts at F and ends at G.



The purpose of Exercises 21 through 24 is for you to learn how to numerically manipulate factorials. (These exercises are repeates of Exercises 37–40 in Chapter 2.) You should answer these questions without using a calculator—otherwise, you are defeating the purpose of the exercise.)

21. **(a)** Given that $10! = 3{,}628{,}800$, find $9!$.

(b) Find $11!/10!$.

(c) Find $11!/9!$.

(d) Find $101!/99!$.

59. Find a Hamilton path in the following graph.