

1. You roll a die 4 times. a) Find N (the number of outcomes in the sample space).
 b) If E is the event: All 4 rolls give the same number, find $Pr(E)$.

$$N = 6^4 = \underline{6 \cdot 6 \cdot 6 \cdot 6}$$

$$Pr(E) = \frac{\# \text{ good outcomes}}{\# \text{ total possibilities}} = \frac{6}{6^4} = \frac{6}{1296}$$

2. There are 5 men and 3 women in line to board a bus. a) In how many ways can they line up?
 b) What is the probability that the first two people in line are men?

a) $N = 8! = \underline{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$

b) $Pr(1^{st} \& 2^{nd} \text{ are men}) = \frac{(5C_2) \cdot 2}{8!} \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$$5C_2 = \frac{5!}{(3! \cdot 2!)} = \frac{5 \cdot 4}{2!} = \frac{5 \cdot 4}{2} = 10$$

$$5P_2 = \frac{5!}{(3!)} = 5 \cdot 4 = 20$$

$$\text{or } \frac{8!}{8!} = 5P_2 \cdot 6! = \frac{8!}{8!} = 1$$

$$= \frac{5 \cdot 4}{8 \cdot 7} = \frac{5 \cdot 4}{8 \cdot 7}$$

3. a) Randomly choose 5 cards from a standard deck. What is the probability that you have no pairs (and therefore no three-of-a-kind or no four-of-a-kind either)?

$$Pr(\text{No pairs}) = \frac{52}{52} \cdot \frac{48}{51} \cdot \frac{44}{50} \cdot \frac{40}{49} \cdot \frac{36}{48}$$

48 out match 1st card

- b) You flip a coin 12 times. What is the probability that you will get exactly 6 heads?

$$Pr(6 \text{ heads}) = \frac{\# \text{ ways to get 6 heads}}{\# \text{ possible}}$$

$$= \frac{12C_6}{2^{12}}$$

pick 6 spots for heads out of 12
 $12C_6$

HH H H H

Evaluate the following: And give an example of what they would count

${}^{80}C_2 = \frac{80!}{(78!)(2!)} = \frac{80 \cdot 79}{2 \cdot 1}$ Pick group of 2
 ${}^{80}P_3 = \frac{80!}{77!} = 80 \cdot 79 \cdot 78$ Order matters
 Pick President VP & Tres.
 same ${}^{80}C_{78}$

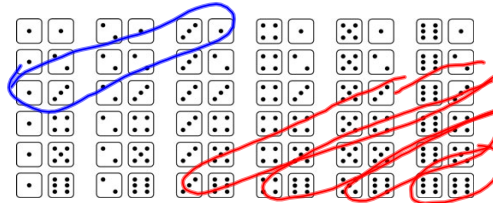
${}^{10}P_4 = 10 \cdot 9 \cdot 8 \cdot 7$

Find the probability of getting a total of 4 when rolling two fair dice.

$Pr(4) = \frac{3}{36}$

Find the probability of getting at least 9 when you roll two fair dice.

$Pr(\geq 9) = \frac{10}{36}$



5. State the Rule, Criterion or Paradox described below:

a) Randy wins an election. A new election is held and the only difference is that some people that had initially voted for Sandy now vote for Randy. Randy should win the new election.

Monotonicity Criterion

b) Randy has 185 pieces of candy to divide among his 12 children. He prepares to divide them according to how many chores each child has done. His son Andy will receive 19 pieces of candy. Just before handing out the candies Randy finds another candy and redistributes them. Now Andy gets just 18 pieces.

Alabama

c) Sandy wins an election. After the election it is determined that another candidate, Mandy, was not a legitimate candidate so her name is removed from the preference schedule and the votes are recounted. Sandy should win the new election.

Independence of Irrelevant Alternatives

states seats population \Leftrightarrow patients

d) There are 3 nursing shifts and 79 nurses to staff these shifts (exactly one shift per nurse). The standard quota for the first shift is 32.8. The first shift should be staffed by either 32 or 33 nurses.

Quota Rule

$$\frac{32.8}{79} = \frac{\text{patients in shift 1}}{\text{all patients}}$$

e) Randy has 185 pieces of candy to divide among his 12 children. He prepares to divide them according to how many chores each child has done. His son Andy will receive 9 pieces of candy and his daughter Mandy will receive 28 pieces. Just before handing out the candies the father Randy determines that Mandy and some other children (but not Andy) have done some additional chores. Randy redistributes the candies and now Andy gets 10 and Mandy gets 27 pieces.

Population Paradox

Number of Voters	21	11	15	13
First Choice	B	A	A	D
Second Choice	D	B	B	C
Third Choice	C	B	C	B
Fourth Choice	A	C	B	A

6. For the preference schedule above, determine the ranking according to the extended Plurality with elimination method.

fewest 1st place votes eliminated then repeat.

A	26
B	21
C	0
D	13

C = 4th

A	26
B	21
D	13

D = 3rd

A	26
<u>B</u>	21+13

*B wins
A 2nd place*

7. For the preference schedule above, determine the ranking according to the recursive Plurality method.

A wins | *rerun electn w/o A*

B	21
C	0
<u>D</u>	39

2nd place *repeat ...*

Number of Voters	21	11	15	13
4th First Choice	B	A	A	D
3rd Second Choice	D	D	D	C
2nd Third Choice	C	B	C	B
1st Fourth Choice	A	C	B	A

For the preference schedule shown above, determine the winner by the method of pairwise comparisons.

Head to Head
A vs B A vs C A vs D B vs C B vs D C vs D
points for head to head win.

For the preference schedule shown above, determine the winner by the method of the Borda count.

Point system
 $A = 21(1) + 11(4) + 15(4) + 13(1) = 138$
D wins.

9. Given the populations for the states A,B,C,D and E below determine the apportionment of 87 seats using Adams' method. Hint: One of the following will work as a modified divisor for this situation: 23.5, 24.1, 24.7, 25.3, 25.9. Write down your successful modified divisor and the apportionments. a) Successful Modified Divisor: b) Apportionments: A = B = C = D = E =

States	Pop.								
A	499	21.3							
B	197	:							
C	457	:							
D	756	:							
E	127	:							

Quota

$$A's \text{ Quota} = 21.3 \frac{Q}{87} = \frac{499}{2036} = \text{Total population} = 2036$$

total seats

Hamilton: Round each Quota down.
Hand out remaining seats in order of highest decimal part.

$$\text{Divisor} = \# \text{ of people per seat} = \frac{2036}{87} = 23.4$$

Adams → increase divisor so quotas go down enough so that when you round up you get right # of seats.
Jefferson → opposite

States	Pop.								
A	499								
B	197								
C	457								
D	756								
E	127								

10. Do the apportionment for problem 9 using Hamilton's Method and 87 seats.

Alabama paradox occurs when an increase in the total number of seats being apportioned in and of itself, forces a state to lose one of its seats

population paradox- when state A loses a seat to state B even though the population of A grew at a higher rate than the population of B.

the new-states paradox- that the addition of a new state with its fair share of seats can, in and of itself, affect the apportionments of other states.

The Quota Rule

No state should be apportioned a number of seats smaller than its lower quota or larger than its upper quota.

round up
round down

The Majority Criterion

If candidate X has a majority of the 1st place votes, then candidate X should be the winner of the election.

more than half
1st place votes

The Condorcet Criterion

If candidate X is preferred by the voters over each of the other candidates in a head-to-head comparison, then candidate X should be the winner of the election.

Pairwise Comparison Method
(Round Robin)

The Monotonicity Criterion

If candidate X is a winner of an election and, in a reelection, the only changes in the ballots are changes that favor X (and only X), then X should remain a winner of the election.

People move
winner up on ballot,
winner should still
win.

The Independence-of-Irrelevant-Alternatives Criterion (IIA)

If candidate X is a winner of an election and in a recount one of the non-winning candidates is removed from the ballots, then X should still be a winner of the election.

Somebody drops out

1. Suppose the value of an investment was decreasing by 12% per year for 5 years. Then it started increasing at 4% for 3 more years.
- a. If the account started at \$20,000. What is the value of the account at the end of the 8 year period?

$$20000 (1 - .12)^5 (1 + .04)^3$$

↑ decrease ↑ increase

$$20000 (.88)^5 (1.04)^3 =$$

- b. Suppose a different account had a value of \$100,000 at the end of the 8 year period, what was the value at the beginning of the 8-year period?

$$A (.88)^5 (1.04)^3 = 100000$$

$$A = 100000 (.88)^{-5} (1.04)^{-3}$$

negative exponent
means back in time

$$= \frac{100000}{(.88)^5 (1.04)^3}$$

same thing.

1. Suppose I buy a microwave from the factory for \$85. How much should I mark-up the price, in order to still make a profit of 20% during the "30% Off Spring Clearance Sale"?

85 increase by 20% then increase by 30%
to set price? ~~30% of wrong amount~~

$$\text{Markup} = \text{Final price}$$
$$85(1.20) = P(1-.30)$$

cost plus profit = price minus discount

$$102 = P(.70)$$

$$\frac{102}{.70} = P$$

$$P = 145.71$$