Sec. 4.1 Exponential Functions

The Exponential Function with Base ‘a’ has domain all real numbers

$$f(x) = a^x$$

Where $a > 0$ and $a \neq 1$

Graphs of Exponentials:

The base ‘a’ can be considered a “multiplying factor” and ‘x’ tells you how many times you have ‘multiplied by a’ (even if ‘x’ is not an integer).

Each time you move 1 unit to the right (increase ‘x’ by 1) the y-value is multiplied by ‘a’.

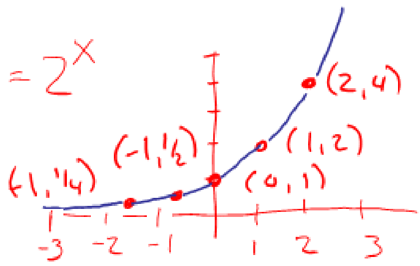
So each time you move 1 unit to the left (decrease ‘x’ by 1) the y-value is divided by ‘a’.

Transformations of Exponential Functions
Start with $y = 2^x$

Transform the graph into the following

$y = -2^x$ $y = 3 - 2^x$

Transformations of Exponential Functions
Start with $y = 2^x$



Transform the graph into the following

$y = 2^{(x-1)}$ $y = \frac{1}{2}(2^x)$

The Natural Exponential Function

- For many reasons, it is often easier to work with exponential functions with the base 'e'.
- $e \approx 2.7182818284\ 5904523536$
- Compare

$f(x) = 2^x$ $f(x) = e^x$ $f(x) = 3^x$

What's so special about 'e'?

- Compound Interest
- Find a formula for how much money you would have in a bank account if you started with $1000 and had 8% interest compounded each year.

$$A(t) = P(1+r)^{(t)}$$

- Here (1.08) is the “multiplying factor” each year.

- What if they computed the interest quarterly?
- What is the “multiplying factor” each quarter? The “quarterly interest rate”?

What’s so special about ‘e’?

- Here is the full formula if they compute interest ‘n’ times per year at ‘r%’ annual interest beginning with P dollars.

$$A(t) = P\left(1+\frac{r}{n}\right)^{(nt)} = P\left(\left(1+\frac{r}{n}\right)^{n}\right)^{t}$$

- So the “annual multiplying factor’ is $\left(1+\frac{r}{n}\right)^{n}$

- And when ‘n’ gets bigger and bigger, this multiplying factor gets closer and closer to e^{r}

$$\left(1+\frac{1}{1}\right)^{1} = 2 \qquad \left(1+\frac{1}{10}\right)^{10} = 2.594$$

$$\left(1+\frac{1}{2}\right)^{2} = 2.25 \qquad \left(1+\frac{1}{1000}\right)^{1000} = 2.718$$

Compounded Interest Formulas

- If the annual interest rate is 'r %' and it is compounded 'n' times each year for 't' years the amount is

$$A(t) = P\left(1+\frac{r}{n}\right)^{(nt)} = P\left(\left(1+\frac{r}{n}\right)^{n}\right)^{t}$$

- If the annual interest rate is 'r %' and it is compounded continuously for 't' years the amount is

$$A(t) = Pe^{rt}$$

- Suppose you invest $1000 in a bank that offers 12% interest compounded monthly. How much interest do you get the first month? How much the second month? How much is your account worth after 1 year? After 10 years?

- Suppose you invest $1000 in a bank that offers 12% interest compounded continuously? How much is your account worth after 1 year? After 10 years?

Homework Problems

Sec. 4.1 # 77

If $3000 is invested at 9% annual interest, find the amount of the investment after 5 years for the following compounding methods.

	Formula	Value
Annual		
Monthly		
Weekly		
Daily		
Continuously		

Homework Problems

Sec. 4.1 # 67

A sky diver jumps from an airplane. The air resistance is proportional to her velocity and the ratio is 0.2. Her downward velocity is given by

$$v(t) = 80\left(1 - e^{-0.2t}\right)$$

Where 't' is in seconds and 'v(t)' is feet/sec.

- Find the initial velocity of the skydiver.
- Find the velocity after 5 seconds.
- Find the velocity after 10 seconds.
- Graph the velocity function.
- What is the terminal velocity of the diver?