

# Spanning subsets of toroidal and Klein bottle embeddings

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## Abstract

Let  $\Phi$  be an embedding of graph  $G$  in a surface  $S$ . If there exists a subset  $K$  of  $S$  bounded by a subgraph of  $G$  which contains all the vertices of  $G$ , then  $K$  is called a *spanning* subset of  $\Phi$ . Examples of spanning subsets include spanning discs, spanning annuli with some number of holes (called planarizing sets in some papers). A spanning subset may provide a simpler structure but still contain enough information to approach certain problems about graphs embedded on surfaces. In this talk, we discuss the existence of various spanning subsets in toroidal and Klein bottle embeddings. For example, we show that for any vertex in a 3-representative toroidal embeddings of a 3-connected graph there exists a spanning cylinder which contains the wheel neighborhood of that vertex.